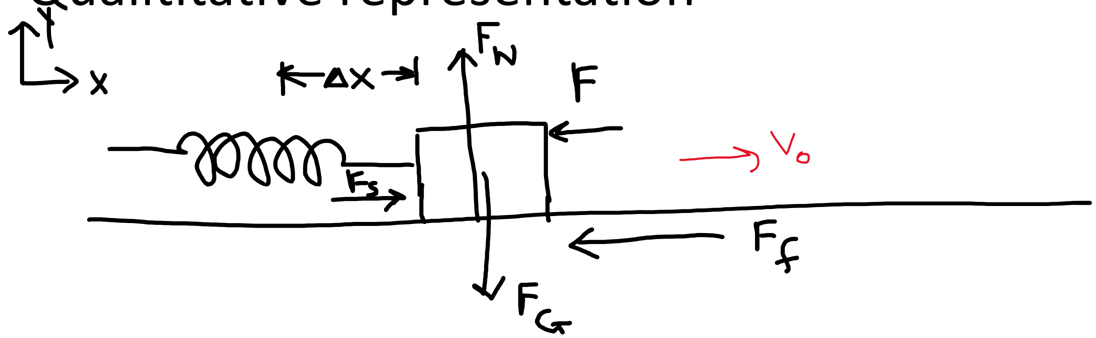
Work-Kinetic energy theorem
and
problem solving

“how do we know what to do” practice

7 April 2020

A 200 g block on a rough surface with coefficient of kinetic friction 0.80 is pushed against a spring with spring constant 500 N/m, compressing the spring 2.0 cm. If the block is released from rest, with what speed is it shot away from the spring?

- Qualititative representation



- Variables and Equations

$m = 200g = 0.2\ kg$ $\mu_k = 0.80$ $k = 500\ N/m$ $\Delta x = 2.0cm = 0.02m$ $v_0 = 0m/s$ $v_f = ?$

$\Sigma F = ma$ $F_G = mg$ $F_{kf} = \mu_k F_N$ $KE = \frac{1}{2}mv^2$ $Work = F\Delta x \cos\theta$

$PE_{spring} = \frac{1}{2}k(\Delta x)^2$ $Work_{non-cons} = \Delta KE + \Delta PE_{spring} + \Delta PE_G$

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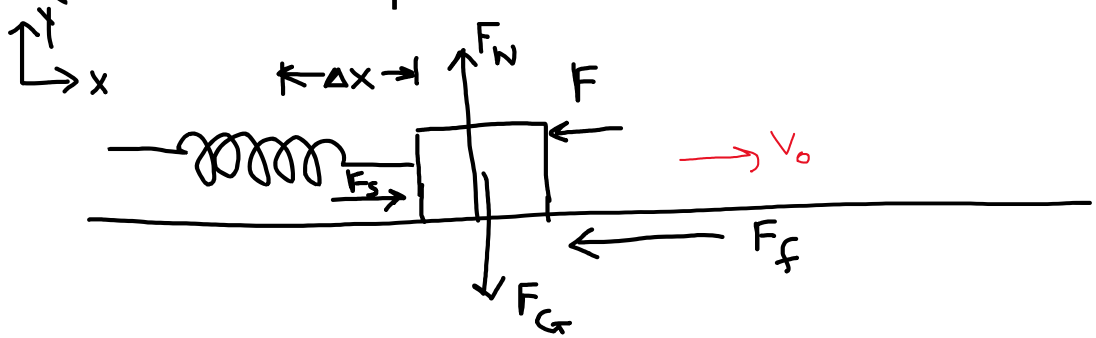

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- How do we know what to do? We need to solve for normal force using F = ma before we can find the frictional force and the work done by friction, so we have to do that first; once we know that, then we work on KE and PE terms to solve

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$F_{kf} = \mu_k F_N = (0.80)(0.200kg)\left(9.8\,\frac{m}{s^2}\right) = 1.568\ N$

What's next???

- Reasonability check?

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$Work = F_{kf}\Delta x\cos\theta = (1.568N)(0.02m)\cos 180 = -0.3136\,J$

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Both the spring force and gravity are conservative, so they both have potential energy changes we can calculate. BUT we don't change height, so the gravitational potential energy change is zero. We can put this into a conservation of energy statement like this:

$Work_{non-cons} = \Delta KE + \Delta PE_{spring} + \Delta PE_G$

$-0.3136\ J = (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2) + (\frac{1}{2}k(\Delta x)_f^2 - \frac{1}{2}k(\Delta x)_0^2) + 0$

How do I fill in details of the conservation of energy equation?

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note: initial KE = 0 since start from rest; final PE_{spring}= 0 since that is where the block leaves the spring

$-0.3136\,J = (\frac{1}{2}(0.2)v_f^2 - 0) + (0 - \frac{1}{2}(500)(0.2)^2) + 0$

Now what? We solve it!

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-0.3136 J = $0.1v_v^2 - 10$

$10 - 0.3136 = 9.6864 = 0.1v_f^2$ $\quad$ $96.864 = v_f^2$ $\quad$ $9.842\frac{m}{s} = v_f$

- Reasonability check? Since this uses a spring as it is de-compressing, we can't really use kinematics to check it (acceleration won't be constant). BUT we know that 10 m/s for a spring-involved motion isn't so suprising.

A 500 kg dragster accelerates from rest to a final speed of 110 m/s in 400 m and encounters an average frictional force of 1200 N. What is its average engine power output if this all takes 7.30 s? Solve using work and energy and power

- Qualitative Representation

- Known and Unknown Variables

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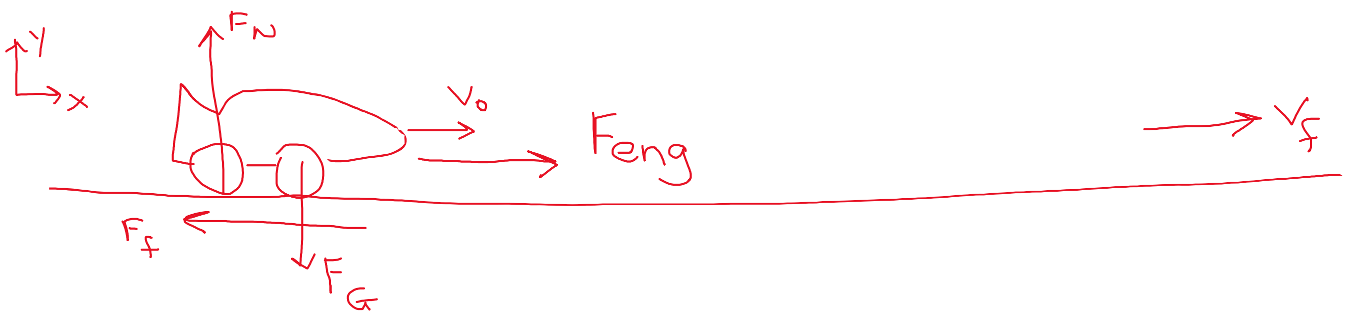


- Known and Unknown Variables

M = 500kg v_0=0 v_f = 110 m/s $\Delta x = 400m$

$F_{kf-average} = 1200N$ $\Delta t = 7.30s$

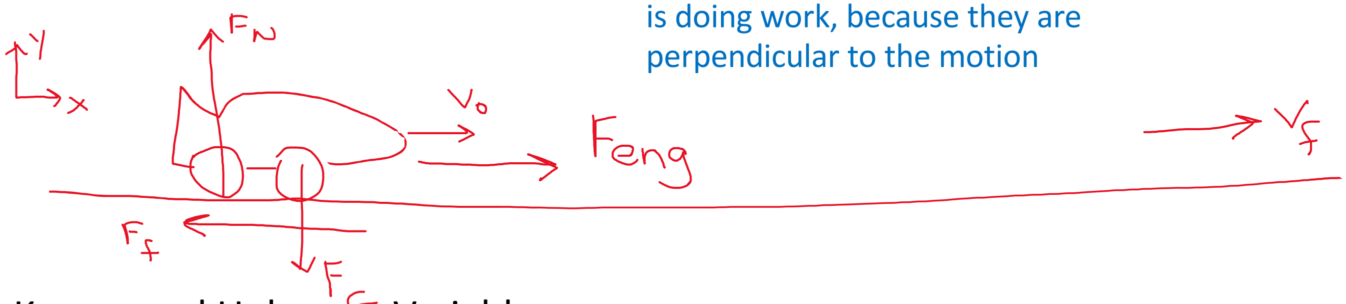
Power_{avg}=?

How do we know what to do here?

A 500 kg dragster accelerates from rest to a final speed of 110 m/s in 400 m and encounters an average frictional force of 1200 N. What is its average engine power output if this all takes 7.30 s? Solve using work and energy and power

- Qualitative Representation

Neither gravity nor the normal force is doing work, because they are perpendicular to the motion



- Known and Unknown Variables

M = 500kg $\quad$ v_0=0 $\quad$ v_f = 110 m/s $\quad$ $\Delta x = 400m$

$F_{kf-average} = 1200N$ $\quad$ $\Delta t = 7.30s$ $\quad$ $Work = F\Delta x\cos\theta$

$Power_{avg}$=? $\quad$ $KE = \frac{1}{2}mv^2$ $\quad$ $Work_{total} = Work_{friction} + Work_{engine}$

Power is defined as work done per unit time, so we need work done by the engine $\quad Power = \frac{work}{time}$

We can use work-kinetic energy theorem to figure out work done by engine $\quad Work_{total} = \Delta KE$

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$$Work_{total} = Work_{friction} + Work_{engine} = \Delta KE$$

$$(F_f \Delta x \cos 180) + Work_{engine} = \frac{1}{2} m v_f^2 - 0$$

$$(1200N)(400m) \cos 180 + Work_{engine} = \frac{1}{2}(500kg)(110m/s)^2$$

$$-480000 + \ Work_{engine} = 3025000$$

$$Work_{engine} = 3505000\, J$$

What do we do now?

A 500 kg dragster accelerates from rest to a final speed of 110 m/s in 400 m and encounters an average frictional force of 1200 N. What is its average engine power output if this all takes 7.30 s? Solve using work and energy and power

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$$Work_{engine} = 3505000\,J$$

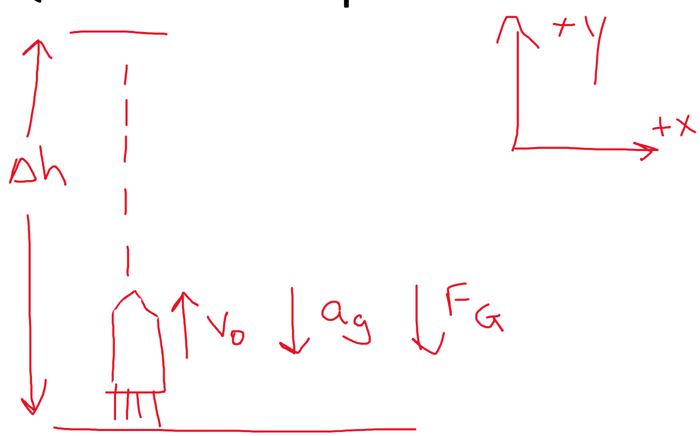
Now we calculate the power of the engine from its' work

$$Power = \frac{work}{time} = \frac{3505000\,J}{7.30\,s} = 480136.98\frac{J}{s} = 480136\,Watts$$

Reasonability Check: 480 KiloWatts is a large amount of energy usage – but we expect that from a car engine.

A 6.25 kg hobby rocket shoots directly up, and reaches a height of 72.3 m. Solve for the initial speed of the rocket using work-energy concepts.

- Qualitative Representation



- Variables

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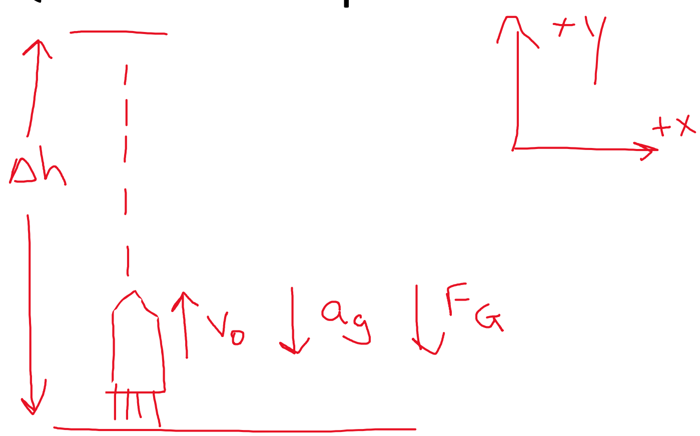

- Variables m = 6.25 kg $\Delta h = 72.3m$ $v_0 =?$ $v_f = 0$

$a = -9.8\ m/s^2$

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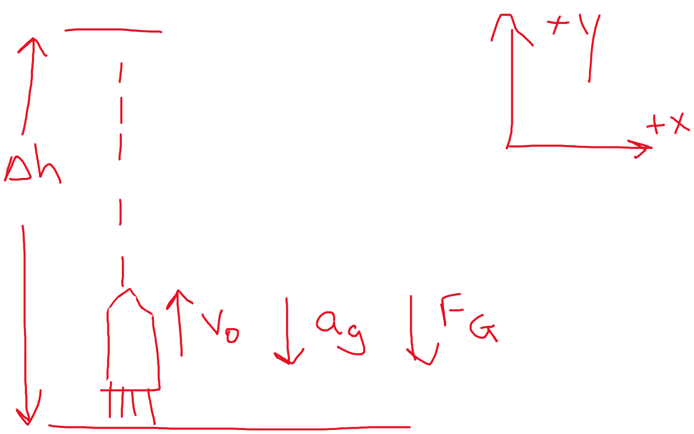

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Problem implies there's no air resistance – so no non-conservative forces. Energy is conserved. We can either use

$$Work_{total} = \Delta KE$$

Or we can use $Work_{non-cons} = \Delta KE + \Delta PE$ where $\text{work}_{\text{non-cons}}=0$

(by the way, the problem told us to use work-energy concepts, so we need to use one of these, not kinematics)

A 6.25 kg hobby rocket shoots directly up, and reaches a height of 72.3 m. Solve for the initial speed of the rocket using work-energy concepts.

- Solution

$$Work_{non-cons} = \Delta KE + \Delta PE = 0$$

$$0 = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right) + (mgh_f - mgh_0)$$

$$0 = \left(\frac{1}{2}(6.25)(0) - \frac{1}{2}(6.25)v_0^2\right) + \left((6.25)(9.8)(72.3m) - (6.25)(9.8)(0)\right)$$

$$0 = (-3.125v_0^2) + \left((6.25)(9.8)(72.3m) - 0\right)$$

$$3.125v_0^2 = 4420.375$$

v_0 = 37.644 m/s

- Reality check: This is about 55 mph, so that's believable for a model rocket. Also – we worked this problem earlier this semester, and could check our answer against the kinematics solution we found before.