Rotational motion work and energy

We have talked about how motion variables have “analogs” for rotational motion.

- What about work and kinetic energy?

- What about momentum?

$$Work = (\Sigma\tau)\theta$$

$$KE_{rot} = \frac{1}{2}I\omega^2$$

$$Work = \Delta KE = \Delta KE_{trans} + \Delta KE_{rot}$$

$$\vec{L} = I\vec{\omega} = \vec{r}\times\vec{p} = rp\sin\theta$$

A ball with an initial velocity of 8.00 m/s rolls up a hill without slipping.
(a) if the ball just slides up the hill without rolling, calculate the vertical height it will reach, by conserving energy.
(b) Treating the ball as a spherical shell that rolls, calculate the vertical height it reaches. *Note: you don't need to know the mass for this.*

Qualitative Representation

Variables (knowns and unknowns) and Equations

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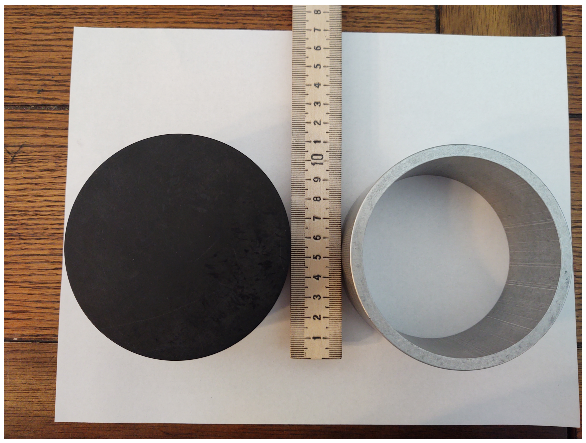
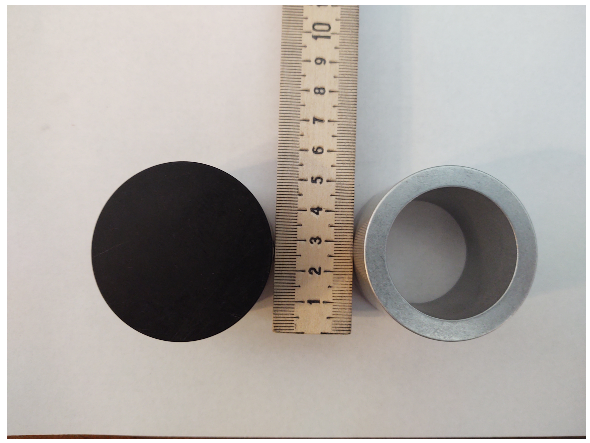
Solutions

Reasonability Check

Now we’re going to race the objects we found the moment of inertia for last time!


	Moment of inertia (results from last class)
Small metal cylindrical ring	$I = (1/2)(r_1^2 + r_2^2) = (1/2)(0.0914\ kg)[(0.019m)^2 + (0.025m)^2] = 4.506 \times 10^{-5}\ kg\ m^2$
Black small solid cylinder	$I = (1/2)mr^2 = (1/2)(0.1092\ kg)(0.025)^2 = 3.4125 \times 10^{-5}\ kg\ m^2$
Large metal cylindrical ring	$I = (1/2)(r_1^2 + r_2^2) = (1/2)(0.219\ kg)[(0.05\ m)^2 + (0.045\ m)^2] = 4.95 \times 10^{-4}\ kg\ m^2$
Large black solid cylinder	$I = (1/2)mr^2 = (1/2)(0.525\ kg)(0.05\ m)^2 = 6.5625 \times 10^{-4}\ kg\ m^2$
Solid white ball	$I = (2/5)mr^2 = (2/5)(.120kg)(0.025m)^2 = 3 \times 10^{-5}\ kg\ m^2$

What will determine which object "wins"?

- Mass
- Moment of inertia
- Final speed (translational)
- Final angular speed (rotational)
- Gravitational potential energy
- Other _________________________

For a ramp of height 0.50 m, we can calculate the gravitational potential energy for each

	mass	Moment of inertia	Initial energy (GPE)
Small metal cylindrical ring	0.914 kg	4.506×10^{-5} kg m^2	
Black small solid cylinder	0.1092 kg	3.4125×10^{-5} kg m^2	
Large metal cylindrical ring	0.219 kg	4.95×10^{-4} kg m^2	
Large black solid cylinder	0.525 kg	6.5625×10^{-4} kg m^2	
Solid white ball	0.120 kg	3×10^{-5} kg m^2	

For a ramp of height 0.50 m, we can calculate the gravitational potential energy for each

	mass	Moment of inertia	Initial energy (GPE)
Small metal cylindrical ring	0.0914 kg	4.506×10^{-5} kg m^2	$Mgh = (0.0914\ kg)(9.81\ m/s^2)(0.5\ m) = 0.448317\ J$
Black small solid cylinder	0.1092 kg	3.4125×10^{-5} kg m^2	$Mgh = 0.535626\ J$
Large metal cylindrical ring	0.219 kg	4.95×10^{-4} kg m^2	$mgh = 1.074195\ J$
Large black solid cylinder	0.525 kg	6.5625×10^{-4} kg m^2	$Mgh = 2.575125\ J$
Solid white ball	0.120 kg	3×10^{-5} kg m^2	$mgh = 0.5886\ J$

Can we conserve energy?

What will the energy be transformed to at the bottom of the ramp?

1. Rotational kinetic energy
2. Translational kinetic energy
3. Both
4. Neither
5. Still gravitational potential energy

For each, set up the equation to conserve energy

$$GPE_i = KE_{rot-f} + KE_{trans-f}$$

	mass	Moment of inertia	Initial energy (GPE)	Final energy (sum of kinetic energies)
Small metal cylindrical ring	0.0914 kg	4.506×10^{-5} kg m^2	0.448317 J	
Black small solid cylinder	0.1092 kg	3.4125×10^{-5} kg m^2	0.535626 J	
Large metal cylindrical ring	0.219 kg	4.95×10^{-4} kg m^2	1.074195 J	
Large black solid cylinder	0.525 kg	6.5625×10^{-4} kg m^2	2.575125 J	
Solid white ball	0.120 kg	3×10^{-5} kg m^2	0.5886 J	

Now plug in details about moment of inertia for each, and solve for speeds at the bottom of the ramp

$$GPE_i = KE_{rot-f} + KE_{trans-f}$$

$$mgh_i = \frac{1}{2}I\omega_f^2 + \frac{1}{2}mv_f^2$$

$$\omega r = v$$

$$mgh_i = \frac{1}{2}I(\frac{v_f}{r})^2 + \frac{1}{2}mv_f^2$$

	mass	Outer radius	Moment of inertia	Initial energy (GPE)	Final energy (sum)	Final speed v	Predicted Order of finish
Small metal cylindrical ring	0.0914 kg	0.025 m	4.506×10^{-5} kg m^2	4.48317 J	0.448317 J		
Black small solid cylinder	0.1092 kg	0.025 m	3.4125×10^{-5} kg m^2	0.535626 J	0.535626 J		
Large metal cylindrical ring	0.219 kg	0.05 m	4.95×10^{-4} kg m^2	1.074195 J	1.074195 J		
Large black solid cylinder	0.525 kg	0.05 m	6.5625×10^{-4} kg m^2	2.575125 J	2.575125 J		
Solid white ball	0.120 kg	0.025 m	3×10^{-5} kg m^2	0.5886 J	0.5886 J		

	mass	Outer radius	Moment of inertia	Initial energy (GPE)	Final energy (sum)	Final speed v	Predicted Order of finish
Small metal cylindrical ring	0.0914 kg	0.025 m	4.506×10^{-5} kg m^2	4.48317 J	0.448317 J	0.95 m/s	
Black small solid cylinder	0.1092 kg	0.025 m	3.4125×10^{-5} kg m^2	0.535626 J	0.535626 J	2.55 m/s	
Large metal cylindrical ring	0.219 kg	0.05 m	4.95×10^{-4} kg m^2	1.074195 J	1.074195 J	2.27 m/s	
Large black solid cylinder	0.525 kg	0.05 m	6.5625×10^{-4} kg m^2	2.575125 J	2.575125 J	2.56 m/s	
Solid white ball	0.120 kg	0.025 m	3×10^{-5} kg m^2	0.5886 J	0.5886 J	2.65 m/s	

Video of the race