ENGR 1100

Week 08- Polynomial I

(Class lecture)

Learning Objectives

Upon completion this module, students will be able to:

1. Understand definition and applications for polynomials
2. Evaluate value of a polynomial in MATLAB
3. Find the roots of a polynomial in MATLAB
4. Compute the addition, Multiplication and Division of polynomials in MATLAB
5. Calculate derivatives of polynomial in MATLAB
6. Compute the integration of Polynomials

What is a Polynomial Function?

A polynomial is a *function* of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1\, x + a_0$$

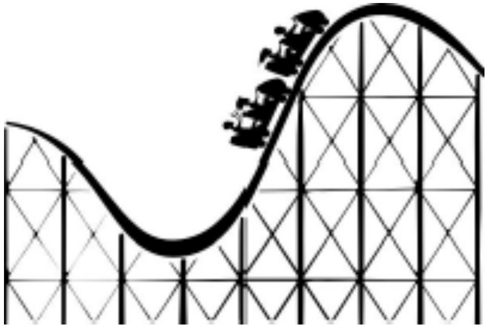
where $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers and *n* is a non-negative integer

- *n* is called the degree or order of the polynomial
- A constant is a polynomial of degree zero

Applications of polynomials

Polynomials are frequently used for problem solving and modelling in science and engineering.

- For example, an engineer designing a roller coaster would use polynomials to model the curves, while a civil engineer would use polynomials to design roads, buildings and other structures.
- Economists use polynomials to model economic growth patterns, and medical researchers use them to describe the behavior of bacterial colonies.



Review of Polynomials

Go over the following resources to review polynomials.

[Khan Academy: Polynomial expressions, equations, & functions.]

The topics that you should focus on are,

1. Polynomial definition and representation.
2. Four basic operations of polynomials: **addition**, **subtraction**, **multiplication** and **division**.
3. Taking **derivatives,** performing **integration,** finding **roots** of polynomials.

Polynomial in MATLAB

Go over the following resources to learn about polynomial in MATLAB.

- MATLAB: Overview of Polynomial
- MATLAB: Create and Evaluate Polynomials
- MATLAB: Roots of Polynomials
- MATLAB:Integrate and Differentiate Polynomials

Representing Polynomial in MATLAB

MATLAB represents a polynomial by a **row vector**

- First vector element is coefficient of x to highest power, i.e., a_n
- Second element is a_{n-1}
- nth element is a_1
- Element $n+1$ is a_0

MATLAB represents a polynomial of degree n by a vector of length n+1

Polynomial	MATLAB representation
$8x + 5$	p = [8 5]
$2x^2 - 4x + 10$	d = [2 –4 10]
$6x^2 - 150$, MATLAB form: $6x^2 + 0x - 150$	h = [6 0 –150]
$5x^5 + 6x^2 - 7x$, MATLAB form: $5x^5 + 0x^4 + 0x^3 + 6x^2 - 7x + 0$	c = [5 0 0 6 –7 0]

Add zeros

Example 1

$$f(x) = x^3 - \frac{23}{2}x^2 + \frac{31}{2}x - 5$$

Evaluate the polynomial when x=2

```
>> p = [ 1 -23/2 31/2 -5 ];
>> polyval( p, 2 )
```

Evaluate the polynomial when x is a vector x=0:0.2:2

```
>> polyval( p, x )
```

Roots of a polynomial

The roots of a polynomial are the values of independent variable that make the polynomial to be zero.

$$f(x) = 0 => x = ?$$

- A polynomial of degree n has exactly n roots, though some may be repeated

```
r = roots(p)
```

r is a column vector with the roots of the polynomial.

p is a row vector with the coefficients of the polynomial.

MATLAB function **`roots`** finds all of the roots of a polynomial

Example 2: Solving Polynomial Equations

A new bakery offers decorated sheet cakes for children's birthday parties and other special occasions. The bakery wants the volume of a small cake to be 351 cubic inches. The cake is in the shape of a rectangular solid. They want the length of the cake to be four inches longer than the width of the cake and the height of the cake to be one-third of the width. What should the dimensions of the cake pan be?

Solution: The volume of a rectangular solid is given by $V = lwh$. We were given that the length must be four inches longer than the width, so we can express the length of the cake as $l = w + 4$. We were given that the height of the cake is one-third of the width, so we can express the height of the cake as $h = \frac{w}{3}$. Let's write the volume of the cake in terms of width of the cake.

$$V = (w+4)(w)\left(\frac{w}{3}\right) = \frac{1}{3}w^3 + \frac{4}{3}w^2$$

Substitute the given volume into this equation, we have

$$351 = \frac{1}{3}w^3 + \frac{4}{3}w^2$$

Rearranging the terms, we have

$$w^3 + 4w^2 - 1053 = 0$$

MATLAB: p=[1 4 0 -1053], roots(p)

➔ the real number solution is *w*=9, so the sheet cake pan should have dimensions 13 inches by 9 inches by 3 inches

Example 3

Find all three roots of the polynomial

$$f(x) = x^3 - \frac{23}{2}x^2 + \frac{31}{2}x - 5$$

```
>> p = [ 1 -23/2 31/2 -5 ];
>> roots( p )
ans =
   10.0000
    1.0000
    0.5000
```

Decreasing order

MATLAB `poly()`

If you know the roots of a polynomial, you can get the polynomial's coefficients with

`p = poly( r )`

where `r` is a vector of roots

```
>> r = [ -1 -1 2 3 ];
>> p = poly( r )
p = 1     -3     -3     7     6
>> roots( p ) % verify that get roots
 ans = 3.0000
    2.0000
       -1.0000
       -1.0000
```

Addition and subtraction of polynomials

To add (subtract) two polynomials, add (subtract) their vectors of coefficients, the vectors should be same length.

- If one vector is shorter, must stick enough zeroes in front of it so same size as longer

Example 4

Add the polynomial

$$f_1(x) = 3x^6 + 15x^5 - 10x^3 - 3x^2 + 15x - 40$$

to the polynomial $f_2(x) = 3x^3 - 2x - 6$

```
>> p1 = [ 3 15 0 -10 -3 15 -40 ];
>> p2short = [ 3 0 -2 -6 ];
>> p2 = [ zeros(1,length(p1)-length(p2short)) p2short ]
p2 =
     0     0     0     3     0    -2    -6
>> p1 + p2
ans =
     3    15     0    -7    -3    13   -46
```

Multiplication of polynomials

Multiply two polynomials with the built--in function **`conv`**, like so:

```
c = conv( a, b )
```

where `a` and `b` are two vectors of polynomial coefficients and `c` is a vector of the coefficients of the product

- `a` and `b` can be different degrees

Example 5

Multiply the polynomial

$$f_{1(x)} = 3x^6 + 15x^5 - 10x^3 - 3x^2 + 15x - 40$$

by the polynomial $f_{2(x)} = 3x^3 - 2x - 6$

```
>> p1 = [ 3 15 0 -10 -3 15 -40 ];
>> p2 = [ 3 0 -2 -6 ];
>> conv( p1, p2 )
ans =
9    45    -6    -78    -99    65    -54    -12    -10    240
```

No need to make them the same length!!

Division of polynomials

Divide two polynomials with the built--in function **deconv**, which has the form

```
[q,r] = deconv(u,v)
```

q is a vector with the coefficients of the quotient polynomial.
r is a vector with the coefficients of the remainder polynomial.

u is a vector with the coefficients of the numerator polynomial.
v is a vector with the coefficients of the denominator polynomial.

Example 6

EXAMPLE

Divide $f_1(x) = 2x^3 + 9x^2 + 7x - 6$ by $x + 3$

```
>> u = [ 2 9 7 -6 ];
>> v = [ 1 3 ];
>> [ q r ] = deconv( u, v )
q =
     2     3    -2
r =
     0     0     0     0
```

so the answer is $2x^2 + 3x - 2$ and a remainder of 0

Example 7

Divide $f_1(x) = 2x^6 - 13x^5 + 75x^3 + 2x^2 - 60$ by $x^2 - 5$

```
>> u = [ 2 -13 0 75 2 0 -60 ];
>> v = [ 1 0 -5 ];
>> [ q r ] = deconv( u, v )
q =
     2    -13      10      10      52
r =
     0      0       0       0       0      50     200
```

Or $2x^4 - 13x^3 + 10x^2 + 10x + 52 + \boxed{\frac{50x+200}{x^2-5}}$

Derivatives of Polynomial

`polyder()` is used to calculate derivative of a polynomial, product of polynomials, or quotient of polynomials

`k = polyder(p)`	Derivative of a single polynomial. `p` is a vector with the coefficients of the polynomial. `k` is a vector with the coefficients of the polynomial that is the derivative.
`k = polyder(a,b)`	Derivative of a product of two polynomials. `a` and `b` are vectors with the coefficients of the polynomials that are multiplied. `k` is a vector with the coefficients of the polynomial that is the derivative of the product.
`[n d] = polyder(u,v)`	Derivative of a quotient of two polynomials. `u` and `v` are vectors with the coefficients of the numerator and denominator polynomials. `n` and `d` are vectors with the coefficients of the numerator and denominator polynomials in the quotient that is the derivative.

Example 8

For example, if $f_1(x) = 3x^2 - 2x + 4$, and $f_2(x) = x^2 + 5$, the derivatives of $3x^2 - 2x + 4$, $(3x^2 - 2x + 4)(x^2 + 5)$, and $\frac{3x^2 - 2x + 4}{x^2 + 5}$ can be determined by:

```
>> f1= 3 -2 4];
>> f2=[1 0 5];

>> k=polyder(f1)

k =
      6    -2

>> d=polyder(f1,f2)

d =
    12    -6    38   -10

>> [n d]=polyder(f1,f2)

n =
      2    22   -10

d =
      1     0    10     0    25
```

Creating the vectors coefficients of f_1 and f_2.

The derivative of f_1 is: $6x - 2$.

The derivative of $f_1 * f_2$ is: $12x^3 - 6x^2 + 38x - 10$.

The derivative of $\frac{3x^2 - 2x + 4}{x^2 + 5}$ is: $\frac{2x^2 + 22x - 10}{x^4 + 10x^2 + 25}$.

Integration of Polynomials

- The integral of any polynomial is the sum of the integral of its terms. For example, the indefinite integral of ax^n is

$$\int ax^n dx = a\frac{x^{n+1}}{n+1} + C$$

- In MATLAB, we can use `polyint()` to integrate polynomial analytically, to generate the coefficients vector of the integral. It takes the coefficients of a polynomial $f(x)$, and gives the coefficients of its integral $\mathrm{g}(x)$.
- How to calculate definite integral of an expression?

Example 9

$$f(x) = 5x^2 - 3x + 2,$$

$$g(x) = \int f(x)dx = \frac{5}{3}x^3 - \frac{3}{2}x^2 + 2x + C$$

MATLAB:

```
a=[5 -3 2];
```

% returns the coefficients of integral with constant of integration as zero.

```
b= polyint(a);
```

% returns the coefficients of integral with constant of integration as 3.

```
b= polyint(a, 3);
```

Recap on Polynomials

- Polynomials A *polynomial* is a function of the form
  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 \, x + a_0$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real and *n* is a non-negative integer

- MATLAB represents a polynomial by a row vector
- Mathematical operations of Polynomials

Operations	
Compute value	polyval(p,x)
roots	r=roots(p) ; p=poly(r)
Add/subtract	p_1+p_2 / p_1 – p_2
Multiply	c=conv(a,b)
Divide	[q,r]=deconv(u,v)

Derivative	k=polyder(p);
	k=polyder(a,b);
	[n, d]=polyder(u,v);

Integration	b=polyint(p)