COMPOUND INTEREST

Math 1001

Quantitative Skills and Reasoning

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COMPOUND INTEREST

- Applying a similar argument for more frequent compounding periods, we derive the following compound amount formula.
- This formula enables to calculate the compound amount for any number of compounding periods per year.

- The compound amount formula is:
  - $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$

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COMPOUND AMOUNT FORMULA

- We use the formula $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$,
  - where *A* is the compound amount,
  - *P* is the amount of money deposited,
  - *r* is the annual interest rate,
  - *n* is the number of compounding periods per year,
  - and *t* is the number of years.

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VALUES OF *N*

- We use the following values for different compounding periods per year:

If interest is compounded:	Then *n* =
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Daily	360

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CALCULATE THE COMPOUND AMOUNT

- Calculate the compound amount when $10,000 is deposited in an account earning 5% interest, compounding quarterly, for 7 years.

- Use the compound amount formula when
  - $P = 10000, \quad r = 0.05, \quad n = 4, \quad t = 7.$
  - $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}} = 10000\left(1 + \frac{0.05}{4}\right)^{4(7)}$
  - $= 10000(1.0125)^{28} = 14159.92$
  - After seven years, the account has $ 14159.92.

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CALCULATE THE COMPOUND AMOUNT

- Recall that future value = compound amount.
- Calculate the future value when $5,000 is deposited in an account earning 9% interest, compounding daily, for 2 years.

- Use the compound amount formula when
  - $P = 5000$, $r = 0.09$, $n = 360$, $t = 2$.
    - $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt} = 5000\left(1 + \frac{0.09}{360}\right)^{360(2)}}$
    - $= 5000(1.00025)^{720} = 5985.95$
    - After two years, the account has $5,985.95.

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CALCULATING COMPOUND INTEREST

- Recall that the formula $I = A - P$ was used in the previous section to find the interest earned on an investment or the interest paid on a loan.
- We can use the same formula after finding A using the compound amount formula.

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CALCULATE THE COMPOUND INTEREST

- How much interest is earned in 3 years on $2,000 deposited in an account paying 4% interest, compounded monthly?

- First, calculate the compound amount with
  - $P = 2000$, $r = 0.04$ $n = 12$ $t = 3$
- $$A = P\left(1 + \frac{r}{n}\right)^{nt} = 2000\left(1 + \frac{0.04}{12}\right)^{12(3)}$$
$$= 2000(1.003333333)^{36} = 2254.54$$

After three years, the account has $2,254.54

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CALCULATE THE COMPOUND INTEREST

- Now, we use this value to calculate the interest earned across the three years:
- $I = A - P$
- $I = 2254.54 - 2000 = 254.54$
- The amount of interest earned is approximately

  $ 254.54

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PRESENT VALUE

- The **present value** of an investment is the original principal invested or the value of the investment before it earns any interest.
- Therefore, it is the principal, *P*, in the compound amount formula.

- **Present value is used to determine how much money must be invested today in order for an investment to have a specific value at a future date.**

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PRESENT VALUE

- The formula for the present value of an investment is found by solving the compound amount formula for *P*:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{P\left(1+\frac{r}{n}\right)^{nt}}{\left(1+\frac{r}{n}\right)^{nt}}$$

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

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PRESENT VALUE

- The present value formula is

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

- where *P* is the original principal invested,
- *A* is the compound amount (the amount you wish to end up with),
- *r* is the annual interest rate,
- *n* is the number of compounding periods per year, and
- *t* is the number of years of the investment.

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CALCULATE PRESENT VALUE

- My sister plans to save for her child's college education. What principal must be deposited by her when her child is born in order to have $41,000 when the child reaches the age of 18? Assume the money earns 7% interest, compounded quarterly. (Round your answer to two decimal places.)

- Use the present value formula, $P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$ with A = 41,000, r = 0.07, n = 4, and t = 18.

- $P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{41{,}000}{\left(1+\frac{0.07}{4}\right)^{4(18)}} = \frac{41{,}000}{(1.0175)^{72}} = 11757.25.$

- The present value, the amount needed now, is $11,757.25.

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EFFECTIVE INTEREST RATE

- When interest is compounded, the annual rate of interest is called the **nominal rate**.
- The **effective rate** is the simple interest rate that would yield the same amount of interest after 1 year.
- When a bank advertises a 8% annual interest rate compounded daily and yielding 8.25%,
  - the nominal interest rate is 8% and
  - the effective rate is 8.25%.

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EFFECTIVE INTEREST RATE

- Consider $100 deposited at 4%, compounded monthly, for 1 year.

  The future value after 1 year is:

- $A = P\left(1 + \frac{r}{n}\right)^{nt} = 100\left(1 + \frac{0.04}{12}\right)^{12(1)} \approx \$104.07$

- The interest earned in 1 year is $104.07 - $100 = $4.07.
- Now consider $100 deposited at an annual simple interest rate of 4.07%:

$$I = Prt = 100(0.0407)(1) = 4.07$$

The interest earned in 1 year is $4.07.

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EFFECTIVE INTEREST RATE

- The interest earned on $100 is the same when it is deposited at 4% compounded monthly as when it is deposited at an annual simple interest rate of 4.07%.
- 4.07% is the effective annual rate of a deposit that earns 4% compounded monthly.

- In the previous example, $100 was used as the principal.

  When we use $100 for *P*, we multiply the interest rate by 100.
- Remember that the interest rate is written as a decimal in the equation $I = Prt$, and a decimal is written as a percent by multiplying by 100.

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EFFECTIVE INTEREST RATE

- When $P = 100$, the digits in the interest earned on the investment ($4.07) are the same as the digits in the effective interest rate (4.07%).
- This is always true, so when we calculate effective interest rate, we will always use $P = 100$.

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CALCULATE EFFECTIVE INTEREST RATE

- A credit union offers a certificate of deposit at an annual interest rate of 3%, compounded daily.
- Find the effective interest rate, rounding to the nearest hundredth of a percent (two decimal places).

- $A = P\left(1 + \frac{r}{n}\right)^{nt} = 100\left(1 + \frac{0.03}{360}\right)^{360(1)} \approx \$103.05$
- I = 103.05 – 100 = $3.05
- The effective interest rate is 3.05%.

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