

PROBABILITY AND ODDS

Math 1001

Quantitative Skills and Reasoning



COLUMBUS STATE
UNIVERSITY

CALCULATING PROBABILITIES WITH DICE

- ▶ Two fair dice are tossed, one after the other. What is the probability that the sum of the pips on the upward faces of the two dice equals 7?
- The dice must be considered as distinct since they are tossed at different times, so there are 36 possible outcomes.
 - Therefore, $n(S) = 36$.
- Let E represent the event that the sum of the pips on the upward faces is 7. These outcomes are $\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$.
 - Therefore, $n(E) = 6$.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)



CALCULATING PROBABILITIES WITH DICE

- The probability that the sum of the pips is 7 is

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$



EMPIRICAL PROBABILITY

- ▶ Probabilities such as those calculated in the preceding examples are sometimes referred to as **theoretical probabilities**.
- ▶ In the previous example about flipping a coin 3 times, we assume that, in theory, we have a perfectly balanced coin, and we calculate the probability based on the fact that each outcome is equally likely.
- ▶ Similarly, we assume the dice in the previous example are equally likely to land with any of the six faces upward.



EMPIRICAL PROBABILITY OF AN EVENT

- If an experiment is performed repeatedly and the occurrence of the event E is observed, the probability $P(E)$ of the event is given by

$$P(E) = \frac{\text{number of times event } E \text{ occurred}}{\text{number of times the experiment was performed}}$$



EMPIRICAL PROBABILITY

- ▶ When a probability is based on data gathered from an experiment, it is called an **experimental probability** or an **empirical probability**.
- ▶ For instance, if we tossed a thumbtack 100 times and recorded the number of times it landed “point up,” the results might be as shown in the table below:

Point Up	16
Side	84
Total	100



EMPIRICAL PROBABILITY

- ▶ From this experiment, the empirical probability of “point up” is

$$\frac{16}{100} = 0.16$$

- ▶ So the probability of a thumbtack landing “point up” is 16%.

Point Up	16
Side	84
Total	100



ODDS OF AN EVENT

- ▶ Let E be an event in a sample space of equally likely outcomes. Then
- ▶ **Odds in favor of E** = $\frac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$
- ▶ **Odds against E** = $\frac{\text{number of unfavorable outcomes}}{\text{number of favorable outcomes}}$



ODDS OF AN EVENT

- ▶ Odds are used to describe the chance of an event occurring. When the odds of an event are written in fractional form, the fraction bar is read as the word “to.”
- ▶ Thus odds of $\frac{1}{2}$ are read as “1 to 2.”
- ▶ We can also write odds of $\frac{1}{2}$ as 1:2, also read as “1 to 2.”



CALCULATE ODDS

- ▶ If a pair of fair dice is rolled once, what are the odds in favor of rolling a sum of 7?
- Let E be the event of rolling a sum of 7.
- We've previously noted that there are 6 favorable outcomes: $\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$
- The remaining 30 possibilities are unfavorable outcomes.
- Thus, the odds in favor of rolling a sum of 7 are $\frac{6}{30}$ (1 to 5).



ODDS AND PROBABILITY

- ▶ Odds express the likelihood of an event and are therefore related to probability.
- ▶ When the odds of an event are known, the probability of the event can be determined.
- ▶ Conversely, when the probability of an event is known, the odds of the event can be determined.



THE RELATIONSHIP BETWEEN ODDS AND PROBABILITY

- ▶ Suppose E is an event in a sample space and that the *odds in favor* of E are $\frac{a}{b}$.
 - ▶ Then $P(E) = \frac{a}{a+b}$.
- ▶ Suppose E is an event in a sample space.
 - ▶ Then the *odds in favor* of E are $\frac{P(E)}{1-P(E)}$.



DETERMINE PROBABILITY FROM ODDS

- ▶ A jewelry box contains 5 white pearl, 2 gold rings and 4 silver rings. What are the odds of drawing a white pearl from the jewelry box?
- ▶ What was the probability of drawing a white pearl from the jewelry box?

- Because the

- **Odds in favor of E** = $\frac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}} = \frac{5}{6}$ means 5:6

- $P(E) = \frac{a}{a+b} = \frac{5}{5+6} = \frac{5}{11}$, which is around a 45.5%.

