PROBABILITY AND ODDS

Math 1001

Quantitative Skills and Reasoning

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INTRODUCTION TO PROBABILITY

- The likelihood of the occurrence of a particular event is described by a number between 0 and 1.
- You can think of this as a percentage ranging from 0% to 100%.
- This number is called the **probability** of the event.
  - An event that is not very likely has a probability close to 0.
  - An event that is almost certain has a probability close to 1.

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INTRODUCTION TO PROBABILITY

- Any event has from a 0% to 100% chance of occurring, probabilities are always between 0 and 1, inclusive.
- If an event MUST occur, its probability is 1.
- If an event CANNOT occur, its probability is 0.

INTRODUCTION TO PROBABILITY

Events that are certain:

- If it is Monday, the probability that tomorrow is Tuesday is certain, therefore the probability is 1.
- If you are eighteen, the probability of you turning nineteen on your next birthday is 1.
- This is a certain event.

Events that are uncertain:

- The probability that tomorrow is Friday if today is Wednesday is 0.
- The probability that you will be eighteen on your next birthday, if you were just born is 0.

INTRODUCTION TO PROBABILITY

- Probabilities can be calculated by considering the outcomes of experiments.
  - Flip a coin and observe the outcome as a head or a tail.
  - Select a restaurant and observe its annual profit or loss.
  - Record the time a person spends at the grocery shop.
- The sample space of an experiment is the set of all possible outcomes of the experiment.

- For example, consider tossing a coin three times and observing the outcome as a head or a tail.
- Using H for head and T for tail, the sample space is:
- S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} – which includes every possible outcome of tossing three coins.

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FIND A SAMPLE SPACE

- A single die is rolled once. What is the sample space for this experiment?

- The sample space is the set of possible outcomes of the experiment:

  S={1,2,3,4,5,6}

EVENTS AND SAMPLE SPACES

- Formally, an event is a subset of a sample space.
- Using the sample space example from the previous example, here are some possible events:

- EX: There are an even number of pips facing up: E_1={2,4,6}
- EX: The number of pips facing up is greater than4: E_2={5,6}

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EVENTS AND SAMPLE SPACES

- The number of pips facing up is less than 20:

S={1,2,3,4,5,6}

Because the number of pips facing up is always less than 20, this event will always occur, thus the event and the sample space are the same.

- The number of pips facing up is greater than 10:

This is an impossible event; the number of pips facing up will never be greater than 10. Therefore, $E_4 = \emptyset$.

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EVENTS AND PROBABILITY

- Outcomes of some experiments are **equally likely**, which means that the chance of any one outcome is just as likely as the chance of another.
- For instance, if 4 balls of the same size but different colors – red, blue, green, and white – are placed in a box and a blindfolded person chooses 1 ball, the chance of choosing a white ball is the same as the chance of choosing any other colored ball.

EVENTS AND PROBABILITY

- In the case of equally likely outcomes, the probability of an event is based on the number of elements in the event and the number of elements in the sample space.
- We will use $n(E)$ to denote the number of elements in the event E and $n(S)$ to denote the number of elements in the sample space S.

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PROBABILITY OF AN EVENT

- For an experiment with sample space S of *equally likely outcomes*, the probability $P(E)$ of an event E is given by

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of elements in } E}{\text{number of elements in sample space } S}$$

PROBABILITY AND DICE

- Because each outcome of rolling a fair die is equally likely, the probability of the events E_1 (number of pips facing up is even) can be determined from the formula for the probability of an event.

$$P(E_1) = \frac{\text{number of elements in } E_1}{\text{total number of elements in the sample space}} = \frac{3}{6} = \frac{1}{2}$$

- The probability of rolling an even number is ½, or 50%.

PROBABILITY AND DICE

Find the probability of:

Rolling a number greater than 4 on a single roll of one die:

$P(E_2) = \frac{2}{6} = \frac{1}{3}$, so the probability of rolling greater than 4 is 33.33%.

Rolling a number less than 20 on a single roll of one die:

$P(E_3) = \frac{6}{6} = 1$, so the probability of rolling less than 20 is 100%.

Rolling a number greater than 10 on a single roll of one die:

$P(E_4) = \frac{0}{6} = 0$, so the probability of rolling greater than 15 is 0%.

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PROBABILITY AND EQUALLY LIKELY OUTCOMES

- A fair coin – one for which is equally likely that heads or tails will result from a single toss – is tossed 3 times.
- What is the probability that 1 head and 2 tails are tossed?

- Recall that we have already determined the sample space for this experiment:
- S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}; $n(S) = 8$
- Now we must determine the elements in the event:
- E = {HTT, THT, TTH}; $n(E) = 3$

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PROBABILITY AND EQUALLY LIKELY OUTCOMES

- A fair coin – one for which is equally likely that heads or tails will result from a single toss – is tossed 3 times.
- What is the probability that 1 head and 2 tails are tossed.
    - The probability can be founding using the equation:

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}.$$

    - The probability is 3/8, or 37.5%

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PROBABILITY

- Is it possible that the probability of some event could be 1.21?
  - No, all probabilities must be between 0 and 1, inclusive.