

MEASURES OF CENTRAL TENDENCY

Math 1001

Quantitative Skills and Reasoning



COLUMBUS STATE
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THE MODE

- ▶ A third type of average is the *mode*.
- ▶ The **mode** of a list of numbers is the number that occurs most frequently.
- Some lists of numbers do not have a mode.
- For instance, in the list 1, 6, 9, 10, 32, 15, 50, each number occurs exactly once.
- Because no number appears more often than the other numbers, there is no mode.



THE MODE

- A list of numerical data can have more than one mode.
- For instance, in the list 4, 2, 2, 7, 8, 2, 4, 3, 8, 3, 7, 3 the number 2 occurs three times and the number 3 occurs three times.
- Each of the other numbers occurs less than three times.
- Thus, 2 and 3 are both modes for the data.



MEASURES OF CENTRAL TENDENCY

NOTES:

- ▶ The mean, the median, and the mode are all averages; however, they are generally not equal.
- ▶ The mean of a set of data is the most sensitive of the averages. A change in any of the numbers changes the mean, and the mean can be changed drastically by changing an extreme value.
- In contrast, the median and the mode of a set of data are usually not changed by changing an extreme value.



MEASURES OF CENTRAL TENDENCY

- ▶ When a data set has one or more extreme values that are very different from the majority of data values, the mean will not necessarily be a good indicator of an average value.



THE WEIGHTED MEAN

- ▶ A value called the *weighted mean* is often used when some data values are more important than others.
- ▶ For instance, many professors determine a student's course grade from the student's tests and the final examination.
- Consider the situation in which a professor counts the final examination score as 2 test scores.
- To find the weighted mean of the student's scores, the professor first assigns a weight to each score.
- In this case the professor could assign each of the test scores a weight of 1 and the final exam score a weight of 2.



THE WEIGHTED MEAN

- ▶ A student with test scores of 62, 72, 75, and a final examination score of 94 has a weighted mean of

$$\frac{(62 \times 1) + (72 \times 1) + (75 \times 1) + (94 \times 2)}{5} = \frac{397}{5} = 79.4$$

- Note that the numerator of the weighted mean is the sum of the products of each test score and its corresponding weight.
- The number 5 in the denominator is the sum of all the weights ($1+1+1+2=5$).
- This procedure can be generalized.



THE WEIGHTED MEAN

- The **weighted mean** of the n numbers $x_1, x_2, x_3, \dots, x_n$ with the respective assigned weights $w_1, w_2, w_3, \dots, w_n$ is

$$\text{Weighted Mean} = \frac{\sum(x \cdot w)}{\sum w}$$

where $\sum(x \cdot w)$ is the sum of the products formed by multiplying each number by its assigned weight, and $\sum w$ is the sum of all the weights.



THE WEIGHTED MEAN

Example: The table shows Tina's one semester course grades. Use the weighted mean formula to find Tina's GPA for this semester.

Course	Grade	Credits
Math	B	4
Biology	D	3
Physics	A	3
Chemistry	C	4

- The B is worth 3 points, with a weight of 4.
- The D is worth 1 point, with a weight of 3.
- The A is worth 4 points, with a weight of 3.
- The C is worth 2 points, with a weight of 4.



FIND A WEIGHTED MEAN

- The sum of all the weights is $4 + 3 + 3 + 4 = 14$.

$$\text{Weighted Mean} = \frac{(3 \times 4) + (1 \times 3) + (4 \times 3) + (2 \times 4)}{14} = \frac{12 + 3 + 12 + 8}{14} =$$

$$\frac{35}{14} = 2.5$$

- Tina's semester GPA is 2.5.

Course	Grade	Credits
Math	B	4
Biology	D	3
Physics	A	3
Chemistry	C	4



DATA AND FREQUENCY DISTRIBUTION

- ▶ Data that has not been organized or manipulated in any manner is called **raw data**.
- ▶ A large collection of raw data may not provide much pertinent information that can be readily observed.
- ▶ A **frequency distribution**, which is a table that lists observed events and the frequency of occurrence of each observed event, is often used to organize raw data.



DATA AND FREQUENCY DISTRIBUTION

- ▶ A teacher records scores on a 20-point quiz for the 30 students in his class. The scores are:

19 20 18 18 17 18 19 17 20 18 20

16 20 15 17 12 18 19 18 19 17 20

18 16 15 18 20 5 0 0

- ▶ These scores could be summarized into a frequency table by grouping like values:



DATA AND FREQUENCY DISTRIBUTION

The Scores are:

19 20 18 18 17 18 19 17 20 18 20
16 20 15 17 12 18 19 18 19 17 20
18 16 15 18 20 5 0 0

Score	Frequency
0	2
5	1
12	1
15	2
16	2
17	4
18	8
19	4
20	6

The formula for a weighted mean can be used to find the mean of the data in a frequency distribution.

The only change is that the weights are replaced with the frequencies.



DATA AND FREQUENCY DISTRIBUTION

- ▶ The formula for a weighted mean can be used to find the mean of the data in a frequency distribution.
- ▶ The only change is that the weights are replaced with the frequencies.
- The numbers in the right-hand column of this table are the frequencies for the numbers in the left-hand column.
- The sum of all the frequencies is 30.
- Mean =
$$\frac{(0 \cdot 2) + (5 \cdot 1) + (12 \cdot 1) + (15 \cdot 2) + (16 \cdot 2) + (17 \cdot 4) + (18 \cdot 8) + (19 \cdot 4) + (20 \cdot 6)}{30}$$
$$= \frac{407}{30} = 16.233$$

