THE COUNTING PRINCIPLE

Math 1001

Quantitative Skills and Reasoning

COUNTING BY USING A TREE DIAGRAM

- A **tree diagram** is another way to organize the outcomes of a multi-stage experiment.
- For Ex: A customer can choose from two Main entrees, three salads , and two desserts for dinner

  How many dinner variations can this customer choose from?

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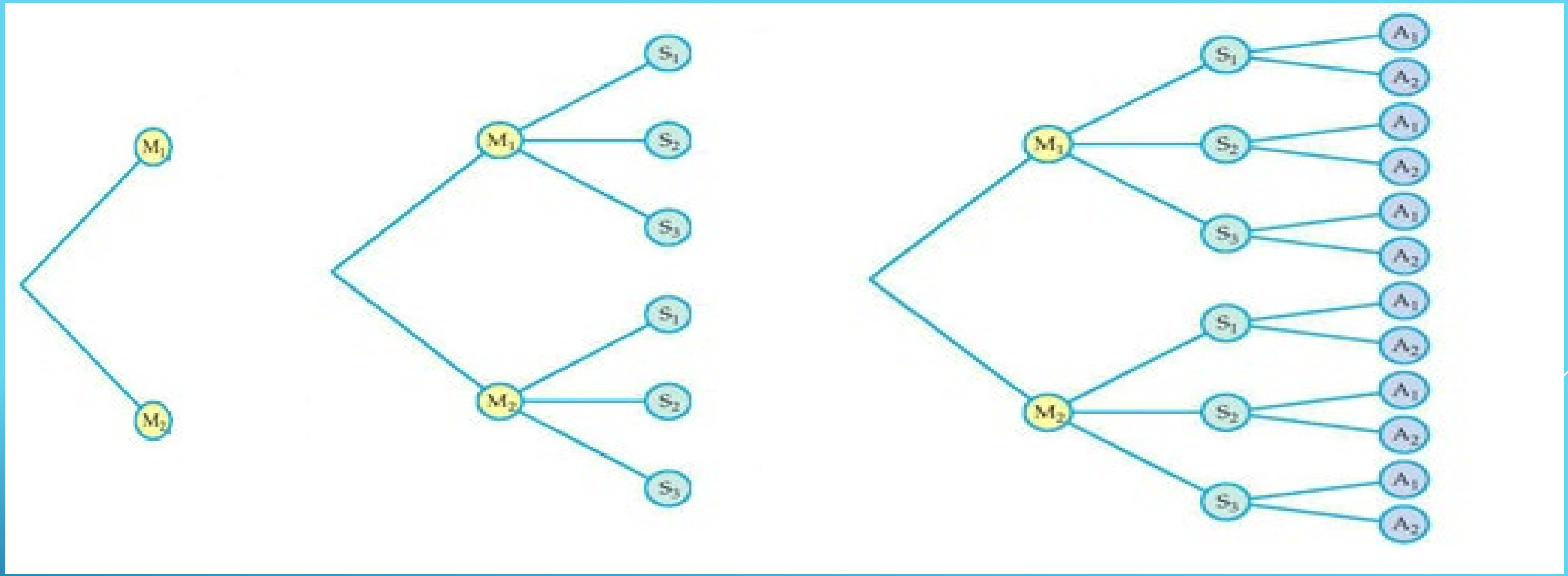
COUNTING BY USING A TREE DIAGRAM

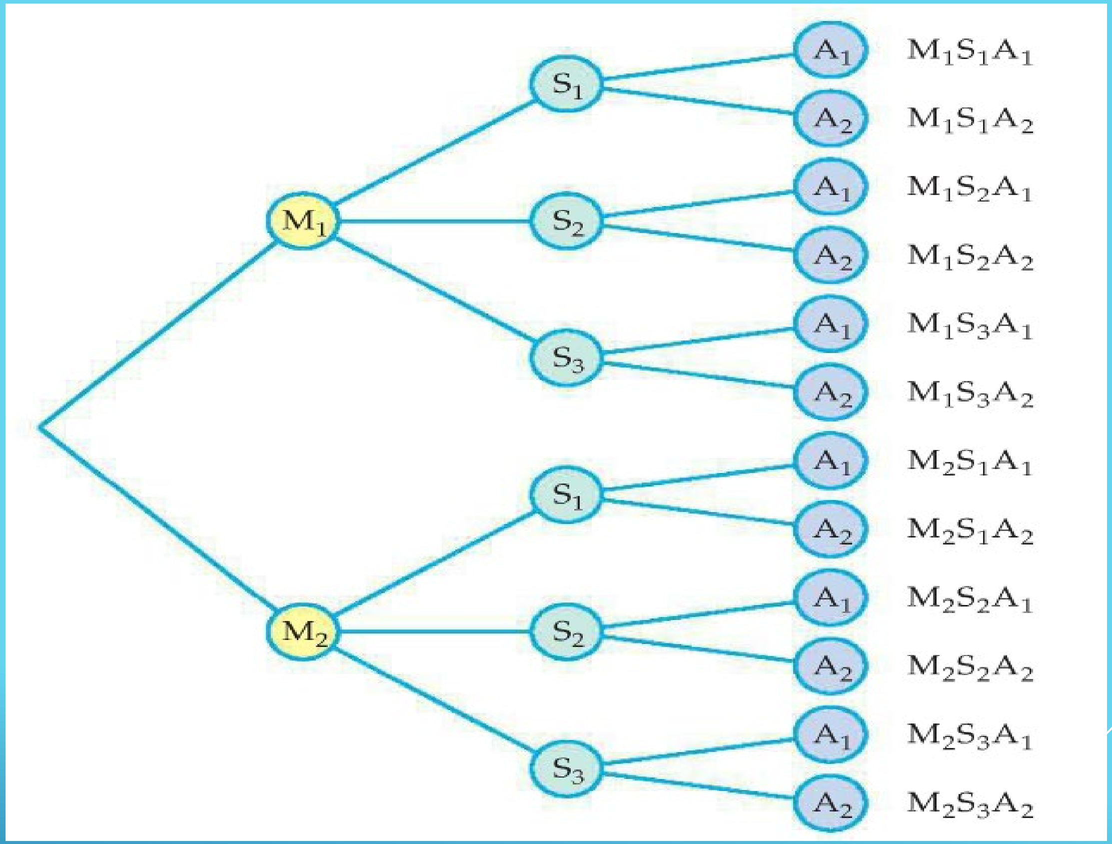
- We can organize the information by letting
  - M1 and M2 represent the two main entrees. S1, S2, and S3 represent the three different salads

    A1 and A2 represent the two different desserts.



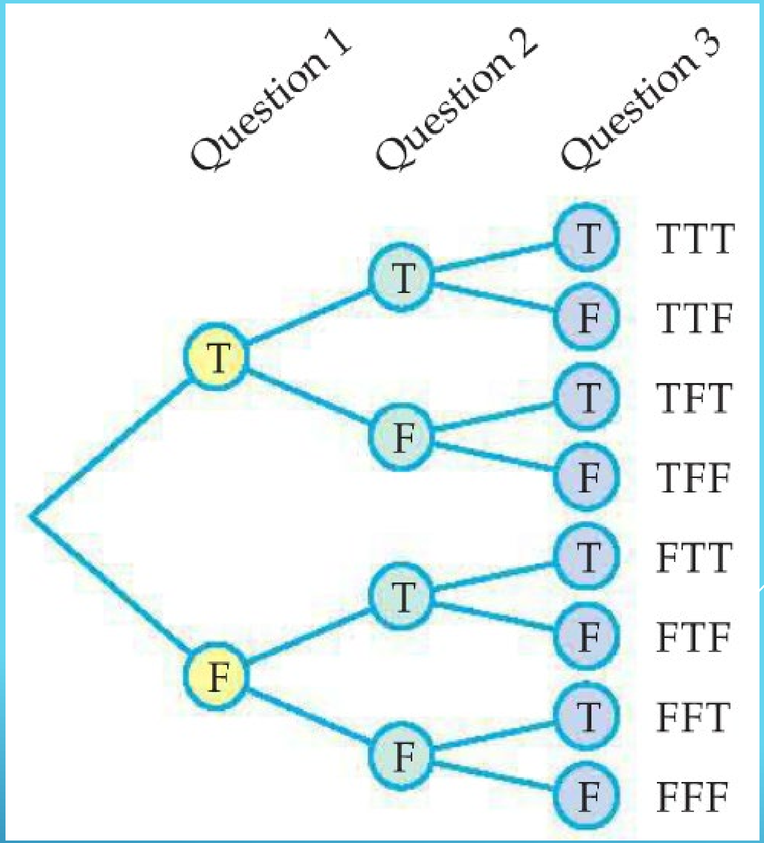
COUNTING BY USING A TREE DIAGRAM

- There are 12 possible dinner variations.



COUNTING USING A TREE DIAGRAM

- A true/false test consists of 12 questions. Draw a tree diagram to show the number of ways to answer the first *three* questions.
  - Note that there are two choices for the first question, two choices for the second question, and two choices for the third question.
  - We let T represent a choice of true, and F represent a choice of false.



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THE COUNTING PRINCIPLE

For each of the previous examples, the possible outcomes were listed and then counted to determine the number of different outcomes.

However, it is not always possible or practical to list and count outcomes.

For example, suppose we tried to draw a tree diagram for all twelve questions in the previous example.

Turns out there are 4096 possible ways to answer all twelve questions. Drawing a tree diagram would QUICKLY get out of hand.

- There is a better way of calculating the number of possibilities, called the **counting principle**.

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THE COUNTING PRINCIPLE

- Consider again the problem of selecting a dinner.
- By using a tree diagram, we listed 12 possible dinners.
- Another way to arrive at this result is to find the product of the number of choices available for Main entrees, salads , and desserts.
  - $\begin{bmatrix} \text{number of} \\ \text{Main } entrees \end{bmatrix} \times \begin{bmatrix} \text{number of} \\ salads \end{bmatrix} \times \begin{bmatrix} \text{number of} \\ \text{desserts} \end{bmatrix} = \begin{bmatrix} \text{number of} \\ \text{dinners} \end{bmatrix}$

$$2 \times 3 \times 2 = 12$$

THE COUNTING PRINCIPLE

- For the example of tossing two dice, there were 36 possible outcomes.
- We can arrive at this result without listing the outcomes by finding the product of the number of possible outcomes of rolling the red die and the number of possible outcomes of rolling the green die.

$$6 \quad \times \quad 6 \quad = \quad 36$$

$$\begin{bmatrix}\text{outcomes}\\ \text{of red die}\end{bmatrix} \times \begin{bmatrix}\text{outcomes}\\ \text{of green die}\end{bmatrix} = \begin{bmatrix}\text{number of}\\ \text{outcomes}\end{bmatrix}$$

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THE COUNTING PRINCIPLE

- This method of determining the number of outcomes of a multi-stage experiment without listing them is called the **counting principle**.

- Let *E* be a multi-stage experiment.
- If $n_1, n_2, n_3, \dots, n_k$ are the number of possible outcomes of each of the *k* stages of *E*, then there are $n_1 \times n_2 \times n_3 \times \cdots \times n_k$ possible outcomes for *E*.

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COUNTING BY USING THE COUNTING PRINCIPLE

- A horse race has 7 horses. How many different ways they can get 1st , 2nd, and 3rd rank assuming there are no ties?

- Any one of the seven horses can be first, so n_1 = 7.
- Because a horse cannot get both first and second rank, there are only six horses that can get second rank, so n_2 = 6.
- Similarly, there are 5 horses that can get third, so n_3 = 5.
- By the counting principle, there are 7(6)(5) = 210 possible ways they can get 1st, 2nd, and 3rd rank.

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COUNTING WITH AND WITHOUT REPLACEMENT

- Consider an experiment in which balls colored red, blue, and green are placed in a box.
- A person reaches into the box and repeatedly pulls out a colored ball, keeping note of the color picked.
- The sequence of colors that can result depends on whether or not the balls are returned to the box after each pick.
- This is referred to as performing the experiment *with replacement* or *without replacement*.

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COUNTING WITH AND WITHOUT REPLACEMENT

- Consider the following two situations:
  - How many three-digit numbers can be formed from the digits 1 through 9 if no digit can be replaced?
  - How many three-digit numbers can be formed from the digits 1 through 9 if a digit can be used repeatedly?

COUNTING WITH AND WITHOUT REPLACEMENT

- How many four-digit numbers can be formed from the digits 1 through 9 if no digit can be replaced?
  - There are nine choices for the first digit (n_1 = 9).
  - Because a digit cannot be repeated, the first digit chosen cannot be used again.
  - Thus, there are only eight choices for the second digit (n_2 = 8).
  - Because neither of the first two digits can be used as the third digit, there are only seven choices for the third digit (n_3 = 7).
  - By the counting principle, there are 9(8)(7)= 504 three-digit numbers in which no digit is repeated.

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COUNTING WITH AND WITHOUT REPLACEMENT

- How many four-digit numbers can be formed from the digits 1 through 9 if a digit can be used repeatedly?
  - There are nine choices for the first digit (n_1 = 9).
  - Because a digit can be used repeatedly, the first digit chosen can be used again.
  - Thus, there are nine choices for the second digit (n_2 = 9).
  - Similarly, there are nine choices for the third digit (n_3 = 9).
  - By the counting principle, there are 9(9)(9) = 729 three-digit numbers in which digits can be used repeatedly.

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COUNTING WITH AND WITHOUT REPLACEMENT

- Does a multi-stage experiment performed *with* replacement generally have more or fewer outcomes than the same experiment performed *without* replacement?
  - MORE!
  - Each stage of an experiment (after the first) performed without replacement will have fewer possible outcomes than the preceding stage.
  - Performed with replacement, each stage of the experiment has the same number of outcomes.

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COUNTING WITH AND WITHOUT REPLACEMENT

- From the letters a, b, c, d, and e, how many three-letter groups can be formed if:
  - A letter can be used more than once?
  - Each letter can be used exactly once?
    - Because each letter can be repeated, there are $5 \times 5 \times 5 = 125$ possible three-letter groups.
    - Because each letter can only be used once, there are $5 \times 4 \times 3 = 60$ three-letter groups in which no letter is repeated.

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