Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

1. Use the **reciprocal** and **quotient** trig identities to evaluate the remaining trigonometric functions.

a. $\tan x = \frac{7}{24}$, $\sec x = \frac{25}{24}$.

$\cot x = \frac{1}{\tan x} = \frac{24}{7}$ $\cos x = \frac{1}{\sec x} = \frac{24}{25}$

b. $\sin x = \frac{\sqrt{3}}{4}$, $\cos x = \frac{\sqrt{13}}{4}$.

$\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{3}/4}{\sqrt{13}/4} = \frac{\sqrt{3}}{\sqrt{13}} = \frac{\sqrt{39}}{13}$ $\cot x = \frac{\cos x}{\sin x} = \frac{\sqrt{13}/4}{\sqrt{3}/4} = \frac{\sqrt{13}}{\sqrt{3}} = \frac{\sqrt{39}}{3}$

$\sec x = \frac{1}{\cos x} = \frac{4}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$ $\csc x = \frac{1}{\sin x} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

2. Use the **Pythagorean** trig identities to find indicated values.

a. $\sin\theta = \frac{1}{3}$, $\cos\theta =$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{1}{9}$

$\cos\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

b. $\sec\theta = 4$. $\tan\theta =$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \tan^2\theta = 4^2$

$\tan^2\theta = 16 - 1$

$\tan\theta = \sqrt{15}$

© 2023. This work is licensed under a CC BY 4.0 license.

c. $\cot\theta = \frac{\sqrt{55}}{3}$. $\csc\theta =$

$$1+\cot^2\theta = \csc^2\theta$$

$$1+\left(\frac{\sqrt{55}}{3}\right)^2 = \csc^2\theta$$

$$1+\frac{55}{9} = \csc^2\theta \Rightarrow \csc\theta = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

3. Use trig identities to find the rest of trigonometric functions given

a) $\cos\alpha = \frac{2}{5}$.

1) $\boxed{\sec\alpha = \frac{1}{\cos\alpha} = \frac{5}{2}}$

2) $\sin^2\alpha + \cos^2\alpha = 1$

$$\sin^2\alpha + \left(\frac{2}{5}\right)^2 = 1$$

$$\sin\alpha = \sqrt{1-\frac{4}{25}}$$

$$\boxed{\sin\alpha = \frac{\sqrt{21}}{5}}$$

3) $1+\tan^2\alpha = \sec^2\alpha$

$$1+\tan^2\alpha = \left(\frac{5}{2}\right)^2$$

$$\tan^2\alpha = \frac{25}{4}-1$$

$$\tan^2\alpha = \frac{21}{4}$$

$$\boxed{\tan\alpha = \frac{\sqrt{21}}{2}}$$

4) $\cot\alpha = \frac{1}{\tan\alpha}$

$$\boxed{\cot\alpha = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}}$$

5) $\csc\alpha = \frac{1}{\sin\alpha}$

$$\boxed{\csc\alpha = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}}$$

© 2023. This work is licensed under a CC BY 4.0 license.

Georgia Gwinnett College
Precalculus
Dr. Nathanson

b) $\cot\beta = 8$

1) $\boxed{\tan\beta = \frac{1}{\cot\beta} = \frac{1}{8}}$

2) $1+\cot^2\beta = \csc^2\beta$

$1+(8)^2 = \csc^2\beta$

$1+64 = \csc^2\beta$

$\boxed{\csc\beta = \sqrt{65}}$

3) $\sin\beta = \frac{1}{\csc\beta}$

$\boxed{\sin\beta = \frac{1}{\sqrt{65}} = \frac{\sqrt{65}}{65}}$

4) $\sin^2\beta + \cos^2\beta = 1$

$\left(\frac{\sqrt{65}}{65}\right)^2 + \cos^2\beta = 1$

$\cos\beta = \sqrt{1 - \frac{1}{65}} = \sqrt{\frac{64}{65}}$

$\boxed{\cos\beta = \frac{8}{\sqrt{65}} = \frac{8\sqrt{65}}{65}}$

5) $\sec\beta = \frac{1}{\cos\beta}$

$\boxed{\sec\beta = \frac{65}{8\sqrt{65}} = \frac{\sqrt{65}}{8}}$

c) $\csc\theta = \frac{7}{3}$.

1) $\boxed{\sin\theta = \frac{1}{\csc\theta} = \frac{3}{7}}$

2) $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{7}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{9}{49}$

$\cos\theta = \sqrt{\frac{40}{49}}$

$\boxed{\cos\theta = \frac{2\sqrt{10}}{7}}$

3) $\boxed{\sec\theta = \frac{1}{\cos\theta} = \frac{7}{2\sqrt{10}} = \frac{7\sqrt{10}}{20}}$

4) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/7}{2\sqrt{10}/7} = \frac{3}{2\sqrt{10}}$

$\boxed{\tan\theta = \frac{3\sqrt{10}}{20}}$

5) $\cot\theta = \frac{1}{\tan\theta} = \frac{20}{3\sqrt{10}} = \frac{20\sqrt{10}}{30}$

$\boxed{\cot\theta = \frac{2\sqrt{10}}{3}}$

© 2023. This work is licensed under a CC BY 4.0 license.