

Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Triangle Inequality Thm:

If $b < a < c$ THEN $B < A < C$

MATH 1113 Precalculus

InClassDrill: OBJ#1113-21

Law of Sines

Name: _____

key

Decide how many triangles are possible in each situation

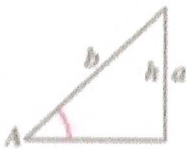


Angle A: acute

Opp & adj $opp < adj$

Opp & ht $opp < ht$

triangles None

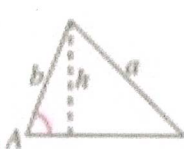


acute

$opp < adj$

$opp = ht$

① right \triangle

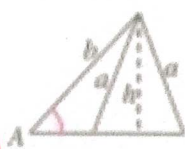


acute

$opp > adj$

$opp > ht$

① oblique \triangle

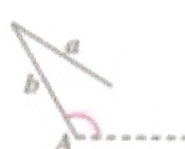


acute

$opp < adj$

$opp > ht$

② oblique \triangle 's



obtuse

$opp < adj$

n/a

None



obtuse

$opp > adj$

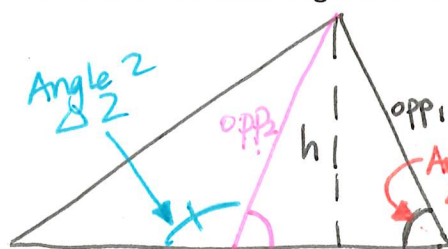
n/a

① oblique \triangle

How to find the height in an oblique triangle:

adjacent side \cdot sine (given angle)

How to find the second angle in a second triangle:

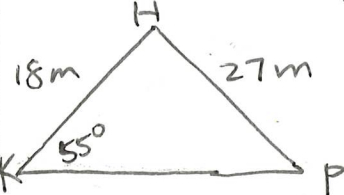


Since $opp_1 = opp_2$
they form
an isosceles
 \triangle with
congruent
base angles

$$\text{Angle } 2 + \text{Angle } 2 = 180^\circ$$

1. In $\triangle KHP$, $m\angle K = 55^\circ$, $p = 18$ m, $k = 27$ m.

Find $m\angle P$



K acute
 $opp > adj$
 $opp > ht$
① oblique \triangle

$$\frac{\sin 55^\circ}{27} = \frac{\sin P}{18} = \frac{\sin ht}{h}$$

$$P = \sin^{-1}\left(\frac{18 \cdot \sin 55^\circ}{27}\right) = 33.1^\circ$$

to finish solving \triangle

$$K = 27 \text{ m} \quad K = 55^\circ$$

$$h = 32.9 \text{ m} \quad H = 91.9^\circ$$

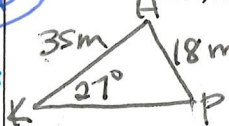
$$p = 18 \text{ m} \quad P = 33.1^\circ$$

$$H = 180^\circ - (55^\circ + 33.1^\circ) = 91.9^\circ$$

$$h = \frac{27 \cdot \sin 91.9^\circ}{\sin 55^\circ} = 32.9$$

2. In $\triangle KHP$, $m\angle K = 27^\circ$, $p = 35$ m, $k = 18$ m.

Find $m\angle P$



K acute
 $opp < adj$
 $ht = 35(\sin 27^\circ) = 15.9$
 $opp > ht$

② oblique \triangle 's

$$\frac{\sin 27^\circ}{18} = \frac{\sin P}{35} = \frac{\sin ht}{h}$$

$$P = \sin^{-1}\left(\frac{35 \cdot \sin 27^\circ}{18}\right) = 62^\circ$$

$$P_2 = 180^\circ - 62^\circ = 118^\circ$$

to finish solving \triangle 's

$$\triangle 1 \quad K = 18 \text{ m} \quad K = 27^\circ$$

$$h = 39.6 \text{ m} \quad H = 91^\circ$$

$$p = 35 \text{ m} \quad P = 62^\circ$$

$$H = 180^\circ - (62^\circ + 27^\circ) = 91^\circ$$

$$h = \frac{18 \cdot \sin 91^\circ}{\sin 27^\circ} = 39.6$$

$$\triangle 2 \quad K = 18 \text{ m} \quad K = 27^\circ$$

$$h = 22.7 \text{ m} \quad H = 35^\circ$$

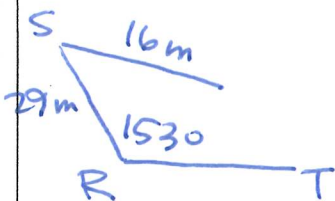
$$p = 35 \text{ m} \quad P = 118^\circ$$

$$h = \frac{18 \cdot \sin 35^\circ}{\sin 27^\circ} = 22.7$$

(H)

3. In ΔRST , $m\angle R = 153^\circ$, $t = 29$ m, $r = 16$ m.

Find $m\angle S$

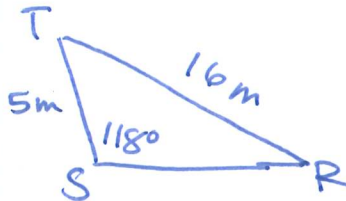


R Obtuse
opp < adj \rightarrow no triangle

(A)

4. In ΔRST , $m\angle S = 118^\circ$, $s = 16$ m, $r = 5$ m.

Find $m\angle R$



S obtuse
opp > adj
 \downarrow
1 oblique Δ

$$\frac{\sin 118^\circ}{16} = \frac{\sin R}{5} = \frac{\sin T}{t}$$

$$R = \sin^{-1}\left(\frac{5 \cdot \sin 118^\circ}{16}\right)$$

$$= 16^\circ$$

finish solving Δ :

r: 5m R: 16° T: $180^\circ - (118 + 16)^\circ = 46^\circ$
s: 16m S: 118° t: $\frac{16 \cdot \sin 46^\circ}{\sin 118^\circ} = 13$
t: 13m T: 46°

(G)

5. In ΔABC , $m\angle A = 64^\circ$, $m\angle B = 98^\circ$, $a = 29$ m.

Find b

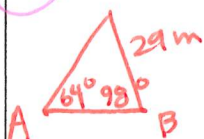
2 angles + 1 side =
no worries!

$$\frac{\sin 64^\circ}{29} = \frac{\sin 98^\circ}{b} = \frac{\sin C}{c}$$

$$b = \frac{29 \cdot \sin 98^\circ}{\sin 64^\circ} = 32$$

$$C = 180^\circ - (64^\circ + 98^\circ) = 18^\circ$$

$$c = \frac{29 \sin 18^\circ}{\sin 64^\circ} = 10$$



finish solving Δ

a: 29m A: 64°

b: 32m B: 98°

c: 10m C: 18°

6. In ΔABC , $m\angle A = 57^\circ$, $c = 35$ m, $a = 33$ m.

Find b



A acute
opp < adj

ht: $35(\sin 57^\circ) = 29.4$ m

opp > ht

2 OBLIQUE Δ s

$\Delta 1$
a: 33m A: 57°
b: 34.1m B: 60°
c: 35m C: 63°

$\frac{\sin 57^\circ}{33} = \frac{\sin C}{35}$
C: $\sin^{-1}\left(\frac{35 \sin 57^\circ}{33}\right) = 63^\circ$

B: $180^\circ - (57^\circ + 63^\circ) = 60^\circ$

b: $\frac{33 \sin 60^\circ}{\sin 57^\circ} = 34.1$

$\Delta 2$
a: 33m A: 57°
b: 4.1m B: 6°
c: 35m C: 117°

C: $180^\circ - 63^\circ$

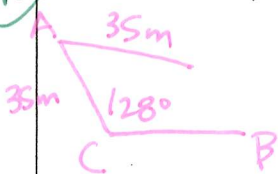
B: $180^\circ - (117^\circ + 57^\circ) = 6^\circ$

b: $\frac{33 \sin 6^\circ}{\sin 57^\circ} = 4.1$

(H)

7. In ΔABC , $m\angle C = 128^\circ$, $b = 35$ m, $c = 35$ m.

Find a



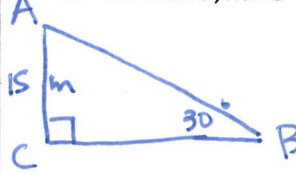
C: obtuse

opp = adj \rightarrow no triangle
(in an obtuse Δ ,
opp must exceed
other sides)

(E)

8. In ΔABC , $m\angle C = 90^\circ$, $m\angle B = 30^\circ$, $b = 15$ m.

Find c



2 angles + 1 side =
no worries!

$$A = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

since rt triangle: use
SOHCAHTOA

$$\sin 30^\circ = \frac{15}{c}$$

$$c = \frac{15}{\sin 30^\circ} = \frac{15}{\frac{1}{2}} = 30$$

Answer bank:

A) 16° B) 62° C) 4 m D) 118° E) 30 m F) 33.1° G) 32 m H) Not a triangle J) 34.1 m