

Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Triangle Inequality Thm:

If $b < a < c$ THEN $B < A < C$

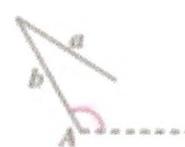
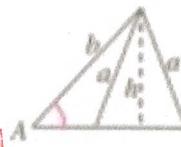
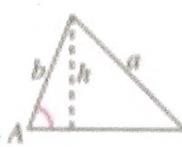
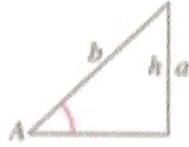
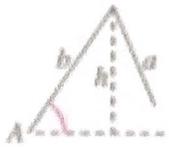
MATH 1113 Precalculus

InClassDrill: OBJ#1113-21

Law of Sines

Name: key

Decide how many triangles are possible in each situation



Angle A: acute

acute

acute

acute

obtuse

obtuse

Opp & adj $opp < adj$

$opp < adj$

$opp > adj$

$opp < adj$

$opp < adj$

$opp > adj$

Opp & ht $opp < ht$

$opp = ht$

$opp > ht$

$opp > ht$

n/a

n/a

triangles None

① right Δ

① oblique Δ

② oblique Δ 's

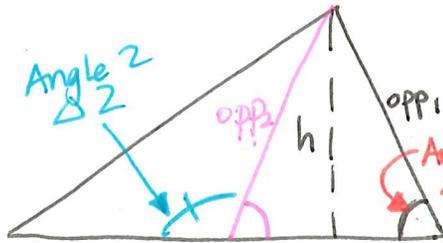
None

① oblique Δ

How to find the height in an oblique triangle:

adjacent side \cdot sine (given angle)

How to find the second angle in a second triangle:

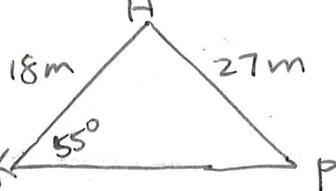


since $opp_1 = opp_2$
they form an isosceles Δ with congruent base angles

Angle 2 + Angle 2 = 180°

1. In ΔKHP , $m\angle K = 55^\circ$, $p = 18$ m, $k = 27$ m.

Find $m\angle P$



$$\frac{\sin 55^\circ}{27} = \frac{\sin P}{18} = \frac{\sin H}{h}$$

$$P = \sin^{-1}\left(\frac{18 \cdot \sin 55^\circ}{27}\right) = 33.1^\circ$$

to finish solving Δ

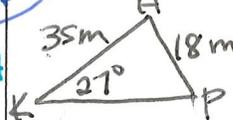
$K = 27$ m $K = 55^\circ$
 $h = 32.9$ m $H = 91.9^\circ$
 $p = 18$ m $p = 33.1^\circ$

$H = 180^\circ - (55^\circ + 33.1^\circ) = 91.9^\circ$

$h = \frac{27 \cdot \sin 91.9^\circ}{\sin 55^\circ} = 32.9$

2. In ΔKHP , $m\angle K = 27^\circ$, $p = 35$ m, $k = 18$ m.

Find $m\angle P$



$$\frac{\sin 27^\circ}{18} = \frac{\sin P}{35} = \frac{\sin H}{h}$$

$$P = \sin^{-1}\left(\frac{35 \cdot \sin 27^\circ}{18}\right) = 62^\circ$$

$$P_2 = 180^\circ - 62^\circ = 118^\circ$$

to finish solving Δ 's

② oblique Δ 's

$\Delta 1$ $\Delta 2$
 $K = 18$ m $K = 27^\circ$ $K = 18$ m $K = 27^\circ$
 $h = 39.6$ m $H = 91^\circ$ $h = 22.7$ m $H = 35^\circ$
 $p = 35$ m $p = 62^\circ$ $p = 35$ m $p = 118^\circ$

$H = 180^\circ - (62^\circ + 27^\circ) = 91^\circ$

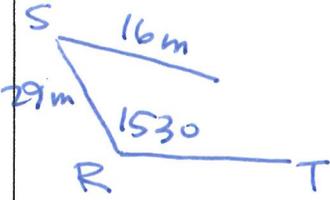
$h = \frac{18 \cdot \sin 91^\circ}{\sin 27^\circ} = 39.6$ m

$H = 180^\circ - (118^\circ + 27^\circ) = 35^\circ$

$h = \frac{18 \cdot \sin 35^\circ}{\sin 27^\circ} = 22.7$

(H)

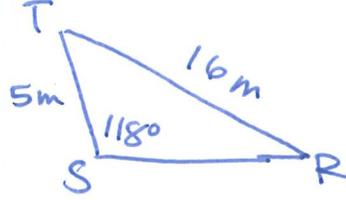
3. In ΔRST , $m\angle R = 153^\circ$, $t = 29\text{ m}$, $r = 16\text{ m}$.
Find $m\angle S$



R Obtuse
opp < adj \rightarrow no triangle

(A)

4. In ΔRST , $m\angle S = 118^\circ$, $s = 16\text{ m}$, $r = 5\text{ m}$.
Find $m\angle R$



S obtuse
opp > adj
 \downarrow
1 oblique Δ

$$\frac{\sin 118^\circ}{16} = \frac{\sin R}{5} = \frac{\sin T}{t}$$

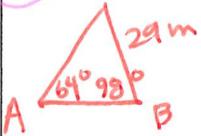
$$R = \sin^{-1}\left(\frac{5 \cdot \sin 118^\circ}{16}\right) = 16^\circ$$

finish solving Δ :

r: 5m R: 16° T: 180° - (118 + 16)° = 46°
s: 16m S: 118° t: $\frac{16 \cdot \sin 46^\circ}{\sin 118^\circ} = 13$

(G)

5. In ΔABC , $m\angle A = 64^\circ$, $m\angle B = 98^\circ$, $a = 29\text{ m}$.
Find b



2 angles + 1 side = no worries!

$$\frac{\sin 64^\circ}{29} = \frac{\sin 98^\circ}{b} = \frac{\sin C}{c}$$

$$b = \frac{29 \cdot \sin 98^\circ}{\sin 64^\circ} = 32\text{ m}$$

$$C = 180^\circ - (64^\circ + 98^\circ) = 18^\circ$$

$$c = \frac{29 \cdot \sin 18^\circ}{\sin 64^\circ} = 10\text{ m}$$

finish solving Δ

a = 29m A = 64°

b = 32m B = 98°

c = 10m C = 18°

6. In ΔABC , $m\angle A = 57^\circ$, $c = 35\text{ m}$, $a = 33\text{ m}$.
Find b



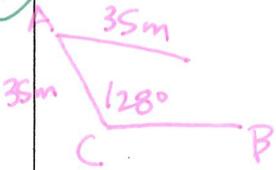
A acute
opp < adj
ht: 35(sin 57°) = 29.4 m
opp > ht
2 OBLIQUE Δ s

$\Delta 1$
a = 33m A = 57°
b = 34.1m B = 60°
c = 35m C = 63°
ht: $35(\sin 57^\circ) = \frac{\sin C}{33} = \frac{\sin c}{35}$
C = $\sin^{-1}\left(\frac{35 \sin 57^\circ}{33}\right) = 63^\circ$
B = 180° - (57° + 63°) = 60°
b = $\frac{33 \sin 60^\circ}{\sin 57^\circ} = 34.1$

$\Delta 2$
a = 33m A = 57°
b = 4.1m B = 6°
c = 35m C = 117°
C = 180° - 63°
B = 180° - (117° + 57°)
b = $\frac{33 \sin 6^\circ}{\sin 57^\circ} = 4.1$

(H)

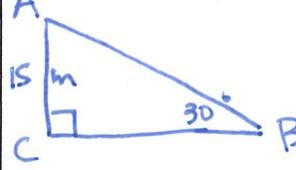
7. In ΔABC , $m\angle C = 128^\circ$, $b = 35\text{ m}$, $c = 35\text{ m}$.
Find a



C: obtuse
opp = adj \rightarrow no triangle
(in an obtuse Δ ,
opp must exceed
other sides)

(E)

8. In ΔABC , $m\angle C = 90^\circ$, $m\angle B = 30^\circ$, $b = 15\text{ m}$.
Find c



2 angles + 1 side = no worries!

$$A = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

since rt triangle: use SOHCAHTOA

$$\sin 30^\circ = \frac{15}{c}$$

$$c = \frac{15}{\sin 30^\circ} = \frac{15}{\frac{1}{2}} = 30\text{ m}$$

Answer bank:

- A) 16° B) 62° C) 4 m D) 118° E) 30 m F) 33.1° G) 32 m H) Not a triangle J) 34.1 m