Sum and Difference Identities

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

Name: _____

Part A: Use a sum or difference identity to condense as a trig function of a single angle; then find the exact value of the expression:

1. $\sin(20°) \cos(80°) - \cos(20°) \sin(80°)$

2. $\cos\left(\frac{5\pi}{12}\right) \cos\left(\frac{7\pi}{12}\right) - \sin\left(\frac{5\pi}{12}\right) \sin\left(\frac{7\pi}{12}\right)$

3. $\dfrac{\tan(40°) - \tan(10°)}{1 + \tan(40°)\tan(10°)}$

4. $\sin\left(\frac{\pi}{18}\right) \cos\left(\frac{5\pi}{18}\right) + \cos\left(\frac{\pi}{18}\right) \sin\left(\frac{5\pi}{18}\right)$

5. $\cos(15°) \cos(75°) + \sin(15°) \sin(75°)$

6. $\dfrac{\tan\left(\frac{\pi}{9}\right) + \tan\left(\frac{5\pi}{36}\right)}{1 - \tan\left(\frac{\pi}{9}\right)\tan\left(\frac{5\pi}{36}\right)}$

Part B: Use a sum or difference identity to find the exact value of the expression:

7. $\sin(30° + 135°)$

8. $\cos\left(\frac{5\pi}{4} - \frac{\pi}{3}\right)$

9. $\tan(30° + 45°)$

10. $\sin\left(\frac{7\pi}{12}\right)$

11. $\tan(195°)$

12. $\cos\left(\frac{\pi}{12}\right)$

13. $\csc\left(\frac{19\pi}{12}\right)$

14. $\cot\left(-\frac{5\pi}{12}\right)$

Part C: Given the trig values of α and β as well as their locations, find the requested sum or differences.

15. $\tan \alpha = -\frac{4}{3}; \frac{\pi}{2} < \alpha < \pi;$ $\cos \beta = \frac{1}{2}, 0 < \beta < \frac{\pi}{2}$

. $\sin(\alpha - \beta)$ $\cos(\beta + \alpha)$ $\tan(\alpha - \beta)$

16. $\sec \alpha = 2; -\frac{\pi}{2} < \alpha < 0;$ $\csc \beta = 3, 0 < \beta < \frac{\pi}{2}$

. $\sin(\alpha - \beta)$ $\cos(\beta + \alpha)$ $\tan(\alpha - \beta)$

Part D: Verify each identity. Work only on one side of the equation.

17. $\tan(2\pi - \beta) = -\tan(\beta)$

18. $\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin(\alpha)$

19. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$

20. $\frac{\cos(\alpha - \beta)}{\sin\alpha\cos\beta} = \cot\alpha + \tan\beta$