Properties of Trig Functions: Period, Odd/Even.

1. Use the unit circle and **periodic properties** of trig functions to find exact values of the following. In each case, break the angle into period and a remainder.

(a) $\cos\dfrac{15\pi}{4} = \cos\dfrac{7\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\dfrac{15\pi}{4} = \underline{2\pi} + \underline{\dfrac{7\pi}{4}}$

(b) $\sin\dfrac{16\pi}{3} = \sin\dfrac{4\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\dfrac{16\pi}{3} = \underline{4\pi} + \underline{\dfrac{4\pi}{3}}$

(c) $\csc\dfrac{43\pi}{6} = \csc\dfrac{7\pi}{6} = -2$

$\dfrac{43\pi}{6} = \underline{6\pi} + \underline{\dfrac{7\pi}{6}}$

$\tan\dfrac{15\pi}{4} = \tan\dfrac{3\pi}{4} = -1$

$\dfrac{15\pi}{4} = \underline{3\pi} + \underline{\dfrac{3\pi}{4}}$

$\cot\dfrac{16\pi}{3} = \cos\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{3}$

$\dfrac{16\pi}{3} = \underline{5\pi} + \underline{\dfrac{\pi}{3}}$

$\tan\dfrac{43\pi}{6} = \tan\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{3}$

$\dfrac{43\pi}{6} = \underline{7\pi} + \underline{\dfrac{\pi}{6}}$

2. Use the unit circle and **even or odd properties** of trig functions to find values of the following.

(a) $\sin\left(-\dfrac{\pi}{3}\right) \overset{odd}{=} -\sin\left(\dfrac{\pi}{3}\right) = -\dfrac{\sqrt{3}}{3}$

(b) $\cos(-150°) \overset{even}{=} \cos(150°) = -\dfrac{\sqrt{3}}{2}$

(c) $\sec\left(-\dfrac{\pi}{6}\right) \overset{even}{=} \sec\left(\dfrac{\pi}{6}\right) = \dfrac{2\sqrt{3}}{3}$

(d) $\tan(-135°) \overset{odd}{=} -\tan(135°) = -(-1) = 1$

(e) $\cot(-90°) \overset{odd}{=} -\cot(90°) = -0 = 0$

(f) $\csc\left(-\dfrac{5\pi}{4}\right) \overset{odd}{=} -\csc\left(\dfrac{5\pi}{4}\right) = -(-\sqrt{2}) = \sqrt{2}$

3. First, use **even or odd** properties, and then **periodic** properties to find the values of the following:

$\sin\left(-\dfrac{17\pi}{4}\right) \overset{odd}{=} -\sin\left(\dfrac{17\pi}{4}\right) = -\sin\left(\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

$\dfrac{17\pi}{4} = 4\pi + \dfrac{\pi}{4}$

$\cos(-495°) \overset{even}{=} \cos(495°) = \cos(135°) = -\dfrac{\sqrt{2}}{2}$

$495° = 360° + 135°$

$\cot\left(-\dfrac{11\pi}{3}\right) \overset{odd}{=} -\cot\left(\dfrac{11\pi}{3}\right) = -\cot\left(\dfrac{2\pi}{3}\right) = -\left(-\dfrac{\sqrt{3}}{3}\right) = \dfrac{\sqrt{3}}{3}$

$\dfrac{11\pi}{3} = 3\pi + \dfrac{2\pi}{3}$

$\csc\left(-\dfrac{29\pi}{6}\right) \overset{odd}{=} -\csc\left(\dfrac{29\pi}{6}\right)$

$\dfrac{29\pi}{6} = 4\pi + \dfrac{5\pi}{6}$

$= -\csc\left(\dfrac{5\pi}{6}\right) = -2$