Sum and Difference Identities

$\sin(a+b)=\sin a\cos b+\cos a\sin b$

$\sin(a-b)=\sin a\cos b-\cos a\sin b$

$\cos(a+b)=\cos a\cos b-\sin a\sin b$

$\cos(a-b)=\cos a\cos b+\sin a\sin b$

$\tan(a+b)=\dfrac{\tan a+\tan b}{1-\tan a\tan b}$

$\tan(a-b)=\dfrac{\tan a-\tan b}{1+\tan a\tan b}$

MATH 1113 Precalculus
InClassDrill: Section 3.1
Sum and Difference Formulas

Name: _____

Part A: Use a sum or difference identity to condense as a trig function of a single angle; then find the exact value of the expression:

1. $\sin(20°)\cos(80°)-\cos(20°)\sin(80°)$

   $\sin(20°-80°)$

   $\sin(-60°)$

   $-\frac{\sqrt{3}}{2}$

2. $\cos\left(\frac{5\pi}{12}\right)\cos\left(\frac{7\pi}{12}\right)-\sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{7\pi}{12}\right)$

   $\cos\left(\frac{5\pi}{12}+\frac{7\pi}{12}\right)$

   $\cos(\pi)$

   -1

3. $\frac{\tan(40°)-\tan(10°)}{1+\tan(40°)\tan(10°)}$

   $\tan(40°-10°)$

   $\tan(30°)$

   $\frac{\sqrt{3}}{3}$

4. $\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{5\pi}{18}\right)+\cos\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)$

   $\sin\left(\frac{\pi}{18}+\frac{5\pi}{18}\right)$

   $\sin\left(\frac{\pi}{3}\right)$

   $\frac{\sqrt{3}}{2}$

5. $\cos(15°)\cos(75°)+\sin(15°)\sin(75°)$

   $\cos(15°-75°)$

   $\cos(-60°)$

   $\frac{1}{2}$

6. $\frac{\tan\left(\frac{\pi}{9}\right)+\tan\left(\frac{5\pi}{36}\right)}{1-\tan\left(\frac{\pi}{9}\right)\tan\left(\frac{5\pi}{36}\right)}$

   $\tan\left(\frac{\pi}{9}+\frac{5\pi}{36}\right)$

   $\tan\left(\frac{\pi}{4}\right)$

   1

Part B: Use a sum or difference identity to find the exact value of the expression:

7. $\sin(30° + 135°)$

$\sin(30°)\cos(135°) + \cos(30°)\sin(135°)$

$\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\frac{-\sqrt{2}+\sqrt{6}}{4}$

8. $\cos\left(\frac{5\pi}{4} - \frac{\pi}{3}\right)$

$\cos\left(\frac{5\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{5\pi}{4}\right)\ sin\left(\frac{\pi}{3}\right)$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$-\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$\frac{-\sqrt{2}-\sqrt{6}}{4}$

9. $\tan(30° + 45°)$

$\frac{\tan(30°)+\tan(45°)}{1-\tan(30°)\tan(45°)}$

$\frac{\frac{\sqrt{3}}{3}+1}{1-\left(\frac{\sqrt{3}}{3}\right)(1)}$

$\frac{3\left(\frac{\sqrt{3}}{3}+1\right)}{3\left[1-\left(\frac{\sqrt{3}}{3}\right)(1)\right]}$

$\frac{\sqrt{3}+3}{3-\sqrt{3}}$

$\left(\frac{\sqrt{3}+3}{3-\sqrt{3}}\right)\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right)$

$2+\sqrt{3}$

10. $\sin\left(\frac{7\pi}{12}\right)$

$\sin\left(\frac{3\pi}{12}\right)\cos\left(\frac{4\pi}{12}\right) + \cos\left(\frac{3\pi}{12}\right)\sin\left(\frac{4\pi}{12}\right)$

$\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

11. $\tan(195°)$

$\tan(45° + 150°)$

$\frac{\tan(45°)+\tan(150°)}{1-\tan(45°)\tan(150°)}$

$\frac{1+\frac{\sqrt{3}}{3}}{1-(1)\left(\frac{\sqrt{3}}{3}\right)}$

$\frac{3\left(1+\frac{\sqrt{3}}{3}\right)}{3\left[1-(1)\left(\frac{\sqrt{3}}{3}\right)\right]}$

$\frac{\sqrt{3}+3}{3-\sqrt{3}}$

$\left(\frac{\sqrt{3}+3}{3-\sqrt{3}}\right)\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right)$

$2+\sqrt{3}$

12. $\cos\left(\frac{\pi}{12}\right)$

$\cos\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)$

$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{3}\right)\ sin\left(\frac{\pi}{4}\right)$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

13. $\csc\left(\frac{19\pi}{12}\right)$

$\frac{1}{\sin\left(\frac{19\pi}{12}\right)}$

$\frac{1}{\sin\left(\frac{4\pi}{12}+\frac{15\pi}{12}\right)}$

$\frac{1}{\sin\left(\frac{\pi}{3}+\frac{5\pi}{4}\right)}$

$\frac{1}{\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{5\pi}{4}\right)+\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{5\pi}{4}\right)}$

$\frac{1}{\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}$

$\frac{1}{-\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}}$

$\frac{4}{-\sqrt{6}-\sqrt{2}}$

$\left(\frac{4}{-\sqrt{6}-\sqrt{2}}\right)\left(\frac{-\sqrt{6}+\sqrt{2}}{-\sqrt{6}+\sqrt{2}}\right)$

$\left(\frac{-4\sqrt{6}+4\sqrt{2}}{4}\right)$

$-\sqrt{6}+\sqrt{2}$

14. $\cot\left(-\frac{5\pi}{12}\right)$

$\frac{1}{\tan\left(-\frac{5\pi}{12}\right)}$

$\frac{1}{\tan\left(-\frac{8\pi}{12}+\frac{3\pi}{12}\right)}$

$\frac{1}{\tan\left(-\frac{2\pi}{3}+\frac{\pi}{4}\right)}$

$\frac{1}{\frac{\tan\left(-\frac{2\pi}{3}\right)+\tan\left(\frac{\pi}{4}\right)}{1-\tan\left(-\frac{2\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}}$

$\frac{1}{\frac{\sqrt{3}+1}{1-(\sqrt{3})(1)}}$

$\frac{1}{\frac{\sqrt{3}+1}{1-\sqrt{3}}}$

$\frac{1-\sqrt{3}}{\sqrt{3}+1}$

$\left(\frac{1-\sqrt{3}}{\sqrt{3}+1}\right)\left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)$

$\sqrt{3}-2$

Part C: Given the trig values of α and β as well as their locations, find the requested sum or differences.

15. $\tan\alpha = -\frac{4}{3}; \frac{\pi}{2} < \alpha < \pi;$ $\cos\beta = \frac{1}{2}, 0 < \beta < \frac{\pi}{2}$

Quad II Quad I

$x = -3, y = 4, r = 5$ $x = 1, y = \sqrt{3}, r = 2$

$\sin\alpha = \frac{4}{5}; cos\,\alpha = -\frac{3}{5}$ $\sin\beta = \frac{\sqrt{3}}{2}; \tan\beta = \sqrt{3}$

. $\sin(\alpha-\beta)$ $\cos(\beta+\alpha)$ $\tan(\alpha-\beta)$

$\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$ $\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ $\frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$

$\left(\frac{4}{5}\right)\left(\frac{1}{2}\right) - \left(-\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$ $\left(-\frac{3}{5}\right)\left(\frac{1}{2}\right) - \left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$ $\frac{-\frac{4}{3}-\sqrt{3}}{1+\left(-\frac{4}{3}\right)(\sqrt{3})}$

$\frac{4}{10} + \frac{3\sqrt{3}}{10}$ $-\frac{3}{10} - \frac{4\sqrt{3}}{10}$ $\frac{3\left[-\frac{4}{3}-\sqrt{3}\right]}{3\left[1+\left(-\frac{4}{3}\right)(\sqrt{3})\right]}$

$\frac{4+3\sqrt{3}}{10}$ $\frac{-3-4\sqrt{3}}{10}$ $\frac{-4-3\sqrt{3}}{3-4\sqrt{3}}$

$\frac{-4-3\sqrt{3}}{3-4\sqrt{3}}$

16. $\sec\alpha = 2; -\frac{\pi}{2} < \alpha < 0;$ $\csc\beta = 3, 0 < \beta < \frac{\pi}{2}$

Quad IV Quad I

$x = 1, y = -\sqrt{3}, r = 2$ $x = 2\sqrt{2}, y = 1, r = 3$

$\sin\alpha = -\frac{\sqrt{3}}{2}; cos\,\alpha = \frac{1}{2}; tan\,\alpha = -\sqrt{3}$ $\sin\beta = \frac{1}{3}; cos\,\beta = \frac{2\sqrt{2}}{3}; \tan\beta = \frac{\sqrt{2}}{4}$

. $\sin(\alpha-\beta)$ $\cos(\beta+\alpha)$ $\tan(\alpha-\beta)$

$\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$ $\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ $\frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$

$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$ $\left(\frac{1}{2}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{3}\right)$ $\frac{-\sqrt{3}-\frac{\sqrt{2}}{4}}{1+(-\sqrt{3})\left(\frac{\sqrt{2}}{4}\right)}$

$-\frac{2\sqrt{6}}{6} - \frac{1}{6}$ $\frac{2\sqrt{2}}{6} + \frac{\sqrt{3}}{6}$ $\frac{4\left[-\sqrt{3}-\frac{\sqrt{2}}{4}\right]}{4\left[1+(-\sqrt{3})\left(\frac{\sqrt{2}}{4}\right)\right]}$

$\frac{-2\sqrt{6}-1}{6}$ $\frac{2\sqrt{2}+\sqrt{3}}{6}$ $\frac{-4\sqrt{3}-\sqrt{2}}{4-\sqrt{6}}$

Part D: Verify each identity. Work only on one side of the equation.

17. $\tan(2\pi - \beta) = -\tan(\beta)$

$$\tan(2\pi - \beta) = -\tan(\beta)$$

$$\frac{\tan(2\pi) - \tan(\beta)}{1 + \tan(2\pi)\tan(\beta)} = -\tan(\beta)$$

$$\frac{0 - \tan(\beta)}{1 + (0)(\tan(\beta))} = -\tan(\beta)$$

$$\frac{-\tan(\beta)}{1} = -\tan(\beta)$$

$$-\tan(\beta) = -\tan(\beta)$$

18. $\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin(\alpha)$

$$\cos\left(\frac{3\pi}{2}\right)\cos(\alpha) - \sin\left(\frac{3\pi}{2}\right)\sin(\alpha) = \sin(\alpha)$$

$$(0)\cos(\alpha) - (-1)\sin(\alpha) = \sin(\alpha)$$

$$\sin(\alpha) = \sin(\alpha)$$

19. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\, cos\beta$

$$\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) + \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) = 2\cos\alpha\, cos\beta$$

$$\cos(\alpha)\cos(\beta) + \cos(\alpha)\cos(\beta) = 2\cos\alpha\, cos\beta$$

$$2\cos(\alpha)\cos(\beta) = 2\cos\alpha\, cos\beta$$

20. $\frac{\cos(\alpha-\beta)}{\sin(\alpha)\cos(\beta)} = \cot(\alpha) + \tan(\beta)$

$$\frac{\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)}{\sin(\alpha)\cos(\beta)} = \cot\alpha + tan\beta$$

$$\frac{\cos(\alpha)\cos(\beta)}{\sin(\alpha)\cos(\beta)} + \frac{\sin(\alpha)\sin(\beta)}{\sin(\alpha)\cos(\beta)} = \cot\alpha + tan\beta$$

$$\frac{\cos(\alpha)}{\sin(\alpha)} + \frac{\sin(\beta)}{\cos(\beta)} = \cot\alpha + tan\beta$$

$$\cot\alpha + tan\beta = \cot\alpha + tan\beta$$