# Chapter 4: Categorical Syllogisms[[1]](#footnote-1)

## I. Standard Form, Mood, and Figure

So far we have just been looking at very short arguments using categorical statements. The arguments just had one premise and a conclusion that was often logically equivalent to the premise. For most of the history of logic in the West, however, the focus has been on arguments that are a step more complicated called categorical syllogisms. A categorical syllogism is a two-premise argument composed of categorical statements. Aristotle began the study of this kind of argument in his book the *Prior Analytics*. This work was refined over the centuries by many thinkers in the Pagan, Christian, Jewish, and Islamic traditions until it reached the form it is in today.

There are actually all kinds of two-premise arguments using categorical statements, but Aristotle only looked at arguments where each statement is in one of the moods A, E, I, or O. The arguments also had to have exactly three terms, arranged so that any two pairs of statements will share one term. Let’s call a categorical syllogism that fits this narrower description an Aristotelian syllogism Here is a typical Aristotelian syllogism using only mood-A sentences:

Premise 1: All mammals are vertebrates.

Premise 2: All dogs are mammals.

Conclusion: Therefore, All dogs are vertebrates.

Notice how the statements in this argument overlap each other. Each statement shares a term with the other two. Premise 2 shares its subject term with the conclusion and its predicate with Premise 1. Thus, there are only three terms spread across the three statements. Aristotle dubbed these the major, middle, and minor premises, but there was initially some confusion about how to define them. In the 6th century, the Christian philosopher John Philoponus, drawing on the work of his pagan teacher Ammonius, decided to arbitrarily designate the major term as the predicate of the conclusion, the minor term as the subject of the conclusion, and the middle term as the one term of the Aristotelian syllogism that does not appear in the conclusion. So, in the argument above, the major term is “vertebrate,” the middle term is “mammal,” and the minor term is “dog.” We can also define the major premise as the one premise in an Aristotelian syllogism that names the major term, and the minor premise as the one premise that names the minor term. So, in the argument above, Premise 1 is the major premise and Premise 2 is the minor premise.

With these definitions in place, we can now define the standard form for an Aristotelian syllogism in logically structured English. Recall that in the Categorical Statements chapter, we started standardizing our language into something we called “logically structured English” in order to remove ambiguity and to make its logical structure clear. The first step was to define the standard form for a categorical statement. Now we do the same thing for an Aristotelian syllogism. We say that an Aristotelian syllogism is in standard form for logically structured English if and only if these criteria have been met: (1) all of the individual statements are in standard form, (2) each instance of a term is in the same format and is used in the same sense, (3) the major premise appears first, followed by the minor premise, and then the conclusion.

Once we standardize things this way, we can actually catalog every possible form of an Aristotelian syllogism. To begin with, each of the three statements can take one of four forms: A, E, I, or O. This gives us 4 × 4 × 4, or 64 possibilities. These 64 possibilities are called the syllogism mood, and we designate it just by writing the three letters of the moods of the statements that make it up. So, the mood of the argument above is simply AAA.

In addition to varying the kind of statements we use in an Aristotelian syllogism, we can also vary the placement of the major, middle, and minor terms. There are four ways we can arrange them that fit the definition of an Aristotelian syllogism in standard form, called the four figures of the Aristotelian Syllogism. The following table demonstrates them all:

|  |  |
| --- | --- |
| Figure 1:  P1: **M** P  P2: S **M**  C: S P | Figure 2:  P1: P **M**  P2: S **M**  C: S P |
| Figure 3:  P1: **M** P  P2: **M** S  C: S P | Figure 4:  P1: P **M**  P2: **M** S  C: S P |

Here P stands for the major term, S for the minor term, and M for the middle. The thing to pay attention to is the placement of the middle terms. In figure 1, the middle terms form a line slanting down to the right. In figure 2, the middle terms are both pushed over to the right. In figure 3, they are pushed to the left, and in figure 4, they slant in the opposite direction from figure 1.

The combination of 64 moods and 4 figures gives us a total of 256 possible Aristotelian syllogisms. We can name them by simply giving their mood and figure. So, this is OAO-3:

Some M are not P.

All M are S.

Some S are not P.[[2]](#footnote-2)

Syllogism OAO-3 is a valid argument. We will be able to prove this with Venn diagrams in the next section. For now, just read it over and try to see intuitively why it is valid. Most of the 256 possible syllogisms, however, are not valid. In fact, most of them, like IIE-2, are quite obviously invalid:

Some P are M.

Some S are M.

No S are P.

Given an Aristotelian syllogism in ordinary English, we can transform it into standard form in logically structured English and identify its mood and figure. Consider the following:

No geckos are cats. I know this because all geckos are lizards, but cats aren’t lizards.

The first step is to identify the conclusion, using the basic skills you acquired earlier. In this case, you can see that “because” is a premise indicator word, so the statement before it, “No geckos are cats,” must be the conclusion.

Step two is to identify the major, middle, and minor terms. Remember that the major term is the predicate of the conclusion, and the minor term is the subject. So here the major term is “cats,” the minor term is “geckos.” The leftover term, “lizards,” must then be the middle term.

We show that we have identified the major, middle, and minor terms by writing a translation key. A translation key is just a list that assigns English phrases or sentences to variable names. For categorical syllogisms, this means matching the English phrases for the terms with the variables S, M, and P.

S: Geckos

M: Lizards

P: Cats

Step three is to write the argument in canonical form using variables for the terms. The last statement, “cats aren’t lizards,” is the major premise, because it has the major term in it. We need to change it to standard form, however, before we substitute in the variables. So first we change it to “No cats are lizards.” Then we write “No S are M.” For the minor premise and the conclusion we can just substitute in the variables, so we get this:

No P are M.

All S are M.

No S are P.

Step four is to identify mood and figure. We can see that this is figure 2, because the middle term is in the predicate of both premises. Looking at the form of the sentences tells us that this is EAE.

## II. Testing Validity

We have seen that there are 256 possible categorical arguments that fit Aristotle’s requirements. Most of them are not valid, and as you probably saw in the exercises, many don’t even make sense. In this section, we will learn to use Venn diagrams to sort the good arguments from the bad. The method we will use will simply be an extension of what we did in the last chapter, except with three circles instead of two.

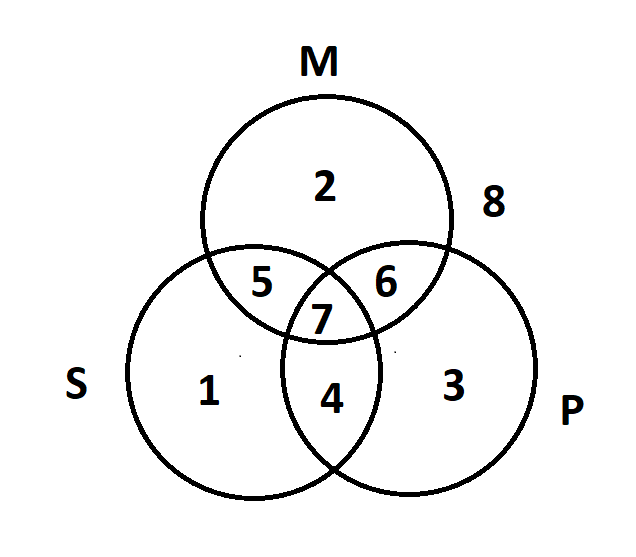
### Venn Diagrams for Single Propositions

In the previous chapter, we drew Venn diagrams with two circles for arguments that had two terms. The circles partially overlapped, giving us four areas, each of which represented a way an individual could relate to the two classes. So area 1 represented things that were S but not P, etc.

.

Two overlapping circles. The left one is labelled S, and the right one is labelled P. In the S circle where it does not overlap with P there is a 1. In the overlap, there is a 2. In the P circle where it does not overlap there is a 3. Outside both circles there is a 4.

Now that we are considering arguments with three terms, we will need to draw three circles, and they need to overlap in a way that will let us represent the eight possible ways an individual can be inside or outside these three classes.



So, in this diagram, area 1 represents the things that are S but not M or P, area 2 represents the things that are M but not S or P, etc.

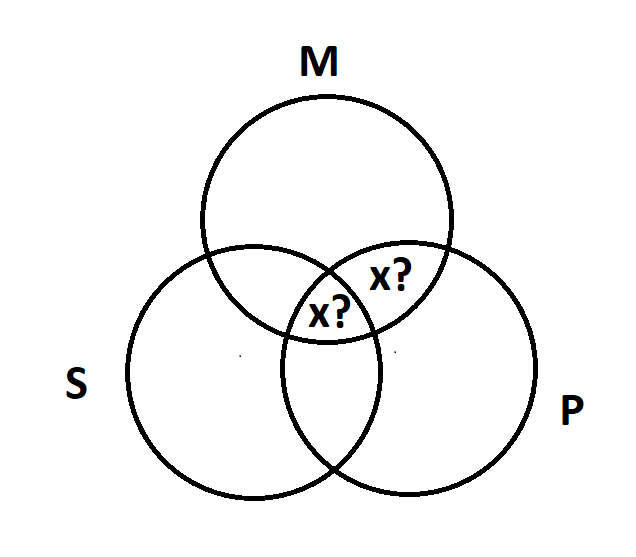
As before, we represent universal statements by filling in the area that the statement says cannot be occupied. The only difference is that now there are more possibilities. So, for instance, there are now four mood-A propositions that can occur in the two premises. The major premise can either be “All P are M” or “All M are P,” and the minor premise can be either “All S are M” or “All M are S.” The Venn diagrams for those four sentences are given in the table below:

|  |  |
| --- | --- |
| Mood A statement, Major Premise  Three overlapping circles, S, M, and P. P is shaded, except for the part where it overlaps with M.  All P are M | Mood A statement, Major Premise  Three overlapping circles, S, M, and P. M is shaded, except for the part where it overlaps with P.  All M are P |
| Mood A statement, Minor Premise  Three overlapping circles, S, M, and P. S is shaded, except for the part where it overlaps with M.  All S are M | Mood A statement, Minor Premise  Three overlapping circles, S, M, and P. M is shaded, except for the part where it overlaps with S.  All M are S |

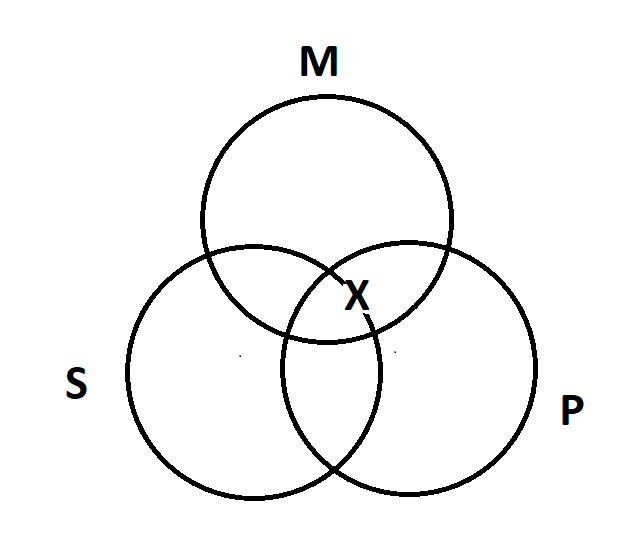
Similarly, there are four mood-E propositions that can occur in the premises of an Aristotelian syllogism: “No P are M,” “No M are P,” “No S are M,” and “No M are S.” And again, we diagram these by shading out overlap between the two relevant circles. In this case, however, the first two statements are equivalent by conversion, as are the second two. Thus, we only have two diagrams to worry about, as seen on the table below:

|  |  |
| --- | --- |
| Mood E statements, major premise  Three overlapping circles, S, M, and P. The diagram is shaded in the two sections where M overlaps with P.  No M are P, or No P are M | Mood E statements, minor premise  Three overlapping circles, S, M, and P. The diagram is shaded in the two sections where M overlaps with S.  No M are S, or no S are M |

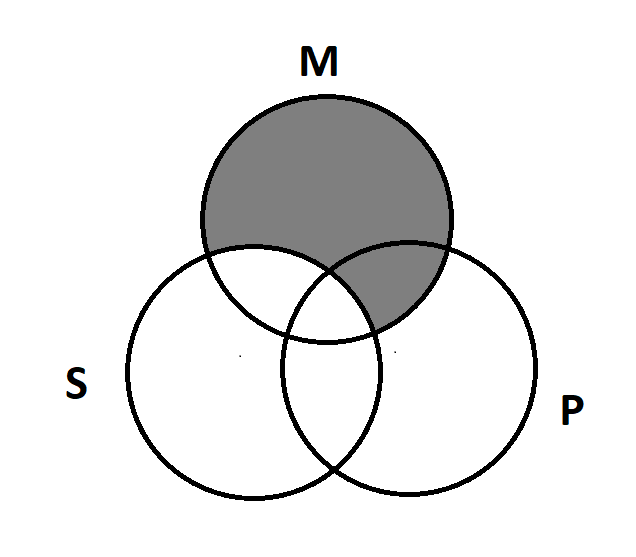
Particular propositions are a bit trickier. Consider the statement “Some M are P.” With a two-circle diagram, you would just put an x in the overlap between the M circle and the P circle. But with the three-circle diagram, there are now two places we can put it. It can go in either area 6 or area 7:



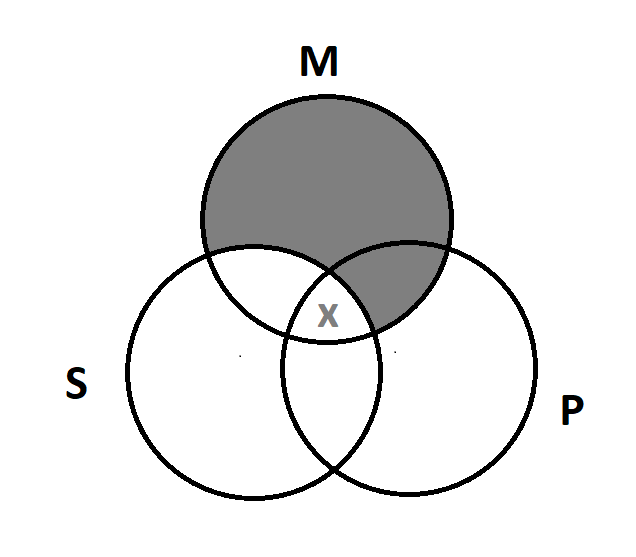
The solution here will be to put the x on the boundary between areas 6 and 7, to represent the fact that it could go in either location:



Sometimes, however, you won’t have to draw the x on a border between two areas, because you will already know that one of those areas can’t be occupied. Suppose, for instance, that you want to diagram “Some M are P,” but you already know that all M are S. You would diagram “All M are S” like this:



Then, when it comes time to add the x for “Some M are P,” you know that it has to go in the exact center of the diagram:



### Venn Diagrams for Full Syllogisms

In the last chapter, we used Venn diagrams to evaluate arguments with single premises. It turned out that when those arguments were valid, the conclusion was logically equivalent to the premise, so they had the exact same Venn diagram. This time we have two premises to diagram, and the conclusion won’t be logically equivalent to either of them. Nevertheless, we will find that for valid arguments, once we have diagrammed the two premises, we will also have diagrammed the conclusion.

First, we need to specify a rule about the order to diagram the premises in: if one of the premises is universal and the other is particular, diagram the universal one first. This will allow us to narrow down the area where we need to put the x from the particular premise, as in the example above where we diagrammed “Some M are P” assuming that we already knew that all M are S.

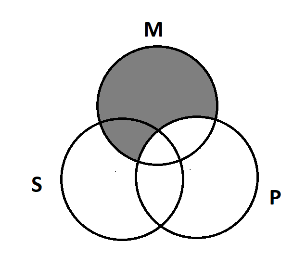
Let’s start with a simple example, an argument with the form AAA-1.

All M are P.

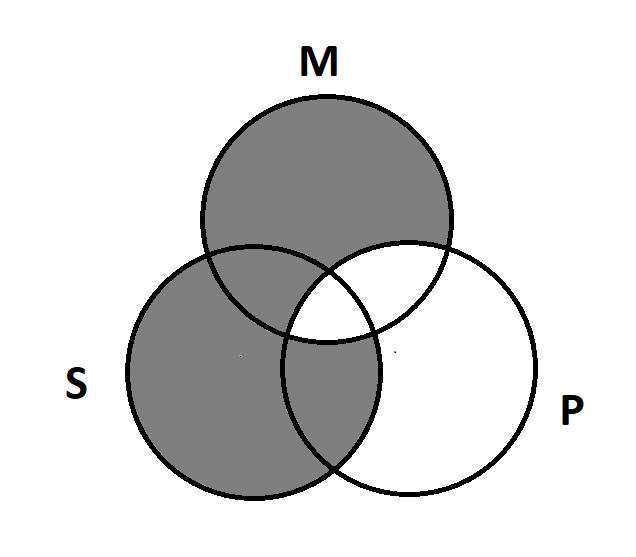
All S are M.

All S are P.

Since both premises are universal, it doesn’t matter what order we do them in. Let’s do the major premise first. The major premise has us shade out the parts of the M circle that don’t overlap the P circle, like this:



The second premise, on the other hand, tells us that there is nothing in the S circle that isn’t also in the M circle. We put that together with the first diagram, and we get this:



From this we can see that the conclusion must be true. All S are P, because the only space left in S is the area in the exact center, area 7.

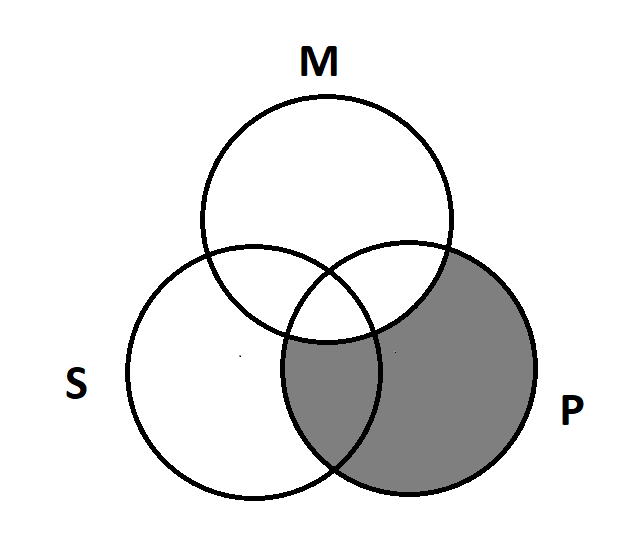
Now let’s look at an argument that is invalid. One of the interesting things about the syllogism AAA-1 is that if you change the figure, it ceases to be valid. Consider AAA-2.

All P are M.

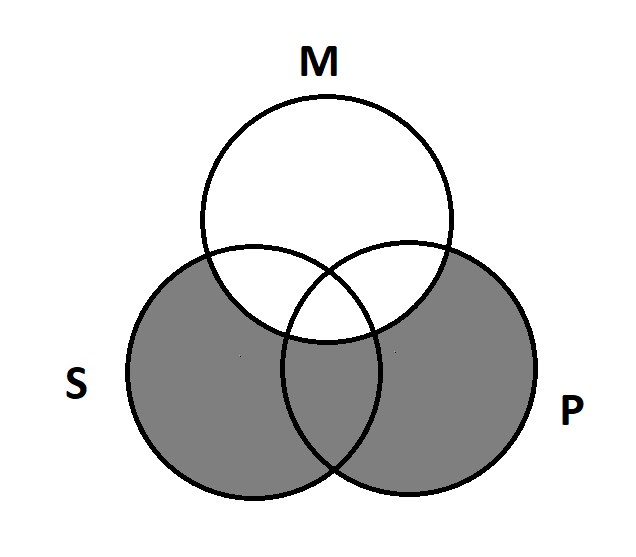
All S are M.

All S are P.

Again, both premises are universal, so we can do them in any order, so we will do the major premise first. This time, the major premise tells us to shade out the part of P that does not overlap M.



The second premise adds the idea that all S are M, which we diagram like this:



Now we ask if the diagram of the two premises also shows that the conclusion is true. Here the conclusion is that all S are P. If this diagram had made this true, we would have shaded out all the parts of S that do not overlap P. But we haven’t done that. It is still possible for something to be in area 5. Therefore, this argument is invalid.

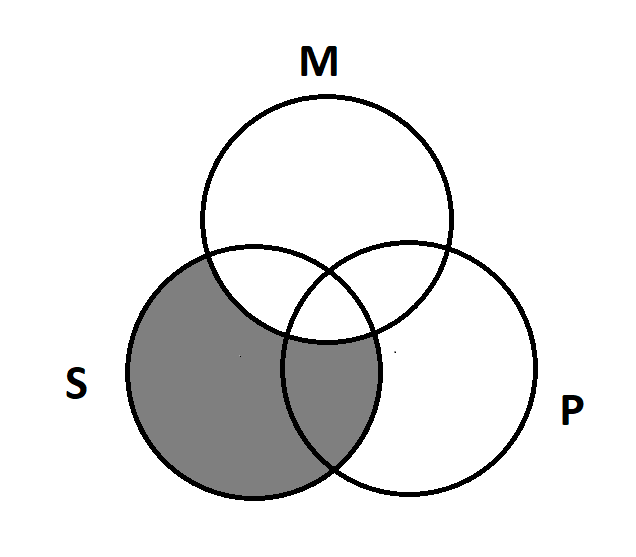
Now let’s try an argument with a particular statement in the premises. Consider the argument IAI-1:

Some M are P.

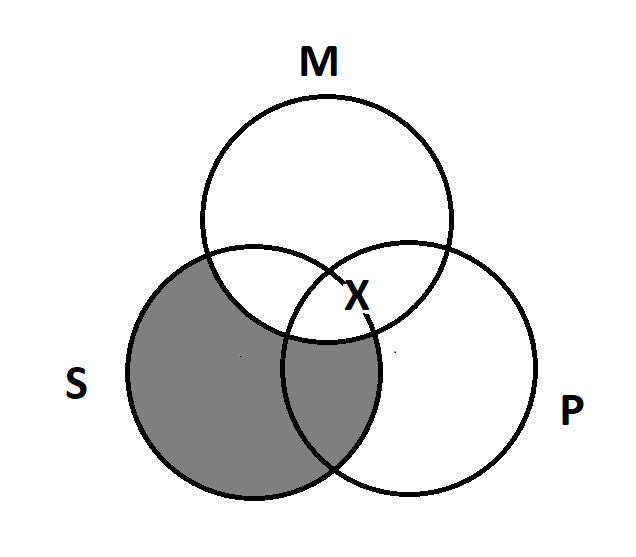
All S are M.

Some S are P.

Here, the second premise is universal, while the first is particular, so we begin by diagramming the universal premise.



Then we diagram the particular premise “Some M are P.” This tells us that something is in the overlap between M and P, but it doesn’t tell us whether that thing is in the exact center of the diagram or in the area for things that are M and P but not S. Therefore, we place the x on the border between these two areas.



Now we can see that the argument is not valid. The conclusion asserts that something is in the overlap between S and P. But the x we drew does not necessarily represent an object that exists in that overlap. There is something out there that could be in area 7, but it could just as easily be in area 6. The second premise doesn’t help us, because it just rules out the existence of objects in areas 1 and 4.

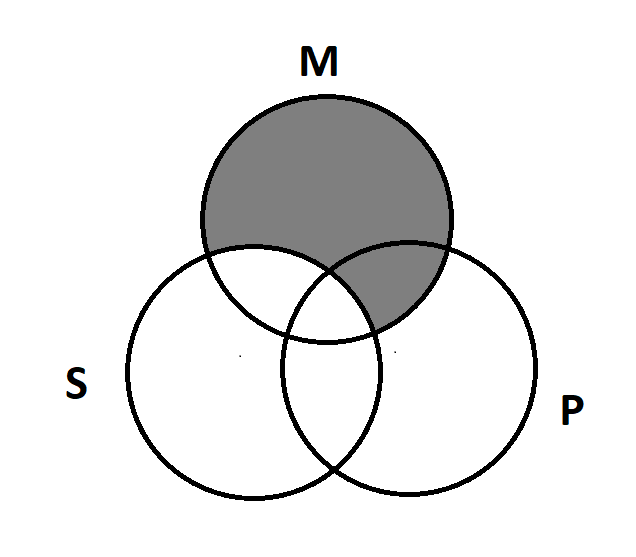
For a final example, let’s look at a case of a valid argument with a particular statement in the premises. If we simply change the figure of the argument in the last example from 1 to 3, we get a valid argument. This is the argument IAI-3:

Some M are P.

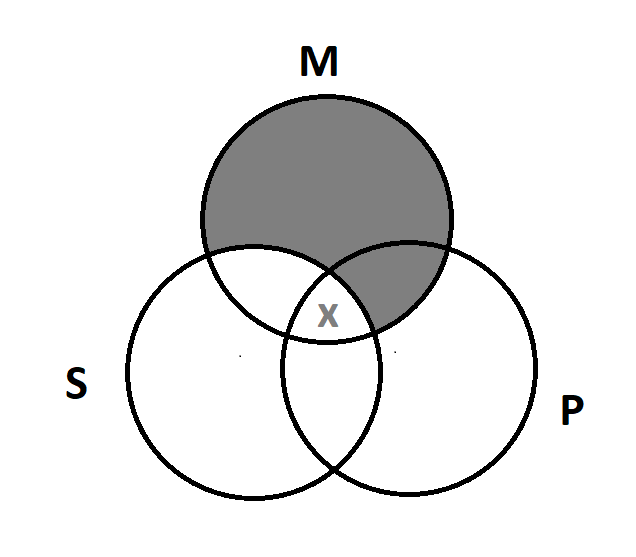
All M are S.

Some S are P.

Again, we begin with the universal premise. This time it tells us to shade out part of the M circle.



But now, since we filled in the parts of M that don’t overlap with S, when we add the particular premise “Some M are P,” we have to put the x in the exact center of the diagram.



And now this time we see that the conclusion, “Some S are P,” has to be true based on the premises, because the X has to be in area 7. So, this argument is valid.

Using this method, we can show that 15 of the 256 possible syllogisms are valid. Remember, however, that the Venn diagram method uses Boolean assumptions about existential import. If you make other assumptions about existential import, you will allow more valid syllogisms, as we will see in the next section. The additional syllogisms we will be able to prove valid in the next section will be said to have conditional validity because they are valid on the condition that the objects talked about in the universal statements actually exist. The 15 syllogisms that we can prove valid using the Venn diagram method have unconditional validity. These syllogisms are given in the following table:

| **Figure 1** | **Figure 2** | **Figure 3** | **Figure 4** |
| --- | --- | --- | --- |
| Barbara (AAA) | Camestres (AEE) | Disamis (IAI) | Calemes (AEE) |
| Celarent (EAE) | Cesare (EAE) | Bocardo (OAO) | Dimatis (IAI) |
| Ferio (EIO) | Festino (EIO) | Ferison (EIO) | Fresison (EIO) |
| Darii (AII) | Baroco (AOO) | Datisi (AII) |  |

The names in this table come from the Christian part of the Aristotelian tradition, where thinkers were writing in Latin. Students in that part of the tradition learned the valid forms by giving each one a name. The vowels in the name represented the mood of the syllogism. So, Barbara has the mood AAA, Fresison has the mood EIO, etc. The consonants in each name were also significant: they related to a process the Aristotelians were interested in called reduction, where arguments in the later figures were shown to be equivalent to arguments in the first figure, which was taken to be more self-evident. We won’t worry about reduction in this textbook, however.

The columns in this table represent the four figures. Syllogisms with the same mood also appear in the same row. So, the EIO sisters—Ferio, Festino, Ferison, and Fresison—fill up row 3. Camestres and Calemes share row 1; Celarent and Cesare share row 2; and Darii and Datisi share row 4.

The names of the valid syllogisms were often worked into a mnemonic poem. The oldest known version of the poem appears in a late 13th century book called Introduction to Logic by William of Sherwood. Below is an image of the oldest surviving manuscript of the poem, digitized by the Bibliothèque Nationale de France (ms. Lat. 16617).

An image from a medieval manuscript, showing a poem in Latin, with names of valid syllogisms included.

## III. Existential Import and Conditionally Valid Forms

In the last section, we mentioned that you can prove more syllogisms valid if you make different assumptions about existential import. Recall that a statement has existential import if, when you assert the statement, you are also asserting the existence of the things the statement talks about. So, if you interpret a mood-A statement as having existential import, it not only asserts “All S is P,” it also asserts “S exists.” Thus, the mood-A statement “All unicorns have one horn” is false, if it is taken to have existential import, because unicorns do not exist. It is probably true, however, if you do not imagine the statement as having existential import. If anything is true of unicorns, it is that they would have one horn if they existed.

We saw in the last chapter that before Boole, Aristotelian thinkers had all sorts of opinions about existential import, or, as they put it, whether a term “supposits.” This generally led them to recognize additional syllogism forms as valid. You can see this pretty quickly if you just remember the traditional square of opposition. The traditional square allowed for many more valid immediate inferences than the modern square. It stands to reason that traditional ideas about existential import will also allow for more valid syllogisms.

Our system of Venn diagrams can’t represent all of the alternative ideas about existential import. For instance, it has no way of representing Ockham’s belief that mood-O statements do not have existential import. Nevertheless, it would be nice if we could expand our system of Venn diagrams to show that some syllogisms are valid if you make additional assumptions about existence.

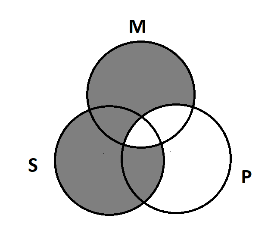
Consider the argument Barbari (AAI-1).

All M are P.

All S are M.

Some S are P.

You won’t find this argument in the list of unconditionally valid forms. This is because under Boolean assumptions about existence it is not valid. The Venn diagram, which follows Boolean assumptions, shows this.



This is essentially the same argument as Barbara, but the mood-A statement in the conclusion has been replaced by a mood-I statement. We can see from the diagram that the mood-A statement “All S are P” is true. There is no place to put an S other than in the overlap with P. But we don’t actually know that the mood-I statement “Some S is P” is true, because we haven’t drawn an x in that spot. Really, all we have shown is that if an S existed, it would be P.

But by the traditional square of opposition (see the previous chapter), we know that the mood-I statement is true. The traditional square, unlike the modern one, allows us to infer the truth of a particular statement given the truth of its corresponding universal statement. This is because the traditional square assumes that the universal statement has existential import. It is really two statements, “All S is P” and “Some S exists.”

Because the mood-A statement is actually two statements on the traditional interpretation, we can represent it simply by adding an additional line to our argument. It is always legitimate to change an argument by making additional assumptions. The new argument won’t have the exact same impact on the audience as the old argument. The audience will now have to accept an additional premise, but in this case all we are doing is making explicit an assumption that the Aristotelian audience was making anyway. The expanded argument will look like this:

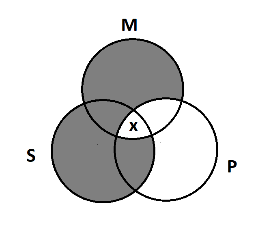
All M are P.

All S are M.

Some S exists.\*

Some S are P

Here the asterisk after “Some S exists” indicates that we are looking at an implicit premise that has been made explicit. Now that we have an extra premise, we can add it to our Venn diagram. Since there is only one place for the S to be, we know where to put our x.



In this argument S is what we call the “critical term.” The critical term is the term that names things that must exist in order for a conditionally valid argument to be actually valid. In this argument, the critical term was S, but sometimes it will be M or P.

We have used Venn diagrams to show that Barbari is valid once you include the additional premise. Using this method we can identify nine more forms, on top of the previous 15, that are valid if we add the right existence assumptions. Below, we repeat the table of unconditionally valid forms, and add a second table, representing the conditionally valid forms, along with what condition must be satisfied to make them valid.

**Unconditionally Valid Forms**

| **Figure 1** | **Figure 2** | **Figure 3** | **Figure 4** | **Condition** |
| --- | --- | --- | --- | --- |
| Barbara (AAA) | Camestres (AEE) | Disamis (IAI) | Calemes (AEE) | None |
| Celarent (EAE) | Cesare (EAE) | Bocardo (OAO) | Dimatis (IAI) | None |
| Ferio (EIO) | Festino (EIO) | Ferison (EIO) | Fresison (EIO) | None |
| Darii (AII) | Baroco (AOO) | Datisi (AII) |  | None |

**Conditionally Valid Forms**

| **Figure 1** | **Figure 2** | **Figure 3** | **Figure 4** | **Condition** |
| --- | --- | --- | --- | --- |
| Barbari (AAI) | Camestros (AEO) |  | Calemos (AEO) | S exists |
| Celaront (EAO) | Cesaro (EAO) |  |  | S exists |
|  |  | Felapton (EAO) | Fesapo (EAO) | M exists |
|  |  | Darapti (AAI) |  | M exists |
|  |  |  | Bamalip (AAI) | P exists |

Thus, we now have an expanded method for evaluating arguments using Venn diagrams. To evaluate an argument, we first use a Venn diagram to determine whether it is unconditionally valid. If it is, then we are done. If it is not, then we see if adding an existence assumption can make it conditionally valid. If we can add such an assumption, add it to the list of premises and put an x in the relevant part of the Venn diagram. If we cannot make the argument valid by including additional existence assumptions, we say it is completely invalid.

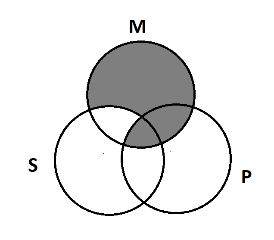
Let’s run through a couple of examples. Consider the argument EAO-3.

No M are P.

All M are S.

Some S are not P.

First, we use the regular Venn diagram method to see whether the argument is unconditionally valid.



We can see from this that the argument is not valid. The conclusion says that some S are not P, but we can’t tell that from this diagram. There are three possible ways something could be S, and we don’t know if any of them are occupied.

Simply adding the premise S exists won’t help us, because we don’t know whether to put the x in the overlap between S and M, the overlap between S and P, or in the area that is just S. Of course, we would want to put it in the overlap between S and M, because that would mean that there is an S that is not P. However, we can’t justify doing this simply based on the premise that S exists.

The premise that P exists will definitely not help us. The P would either go in the overlap between S and P or in the area that is only P. Neither of these would show “Some S is not P.”

The premise “M exists” does the trick, however. If an M exists, it has to also be S but not P. And this is sufficient to show that some S is not P. We can then add this additional premise to the argument to make it valid.

No M are P.

All M are S.

M exists.\*

Some S are not P.

Checking it against the tables of unconditionally and conditionally valid forms, we see that we were right: this is a conditionally valid argument named Felapton.

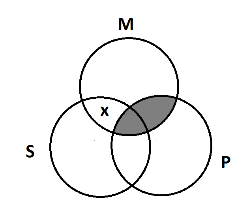
Now consider the argument EII-3:

No M are P.

Some M are S.

Some S are P.

First, we need to see if it is unconditionally valid. So, we draw the Venn diagram.



The conclusion says that some S are P, but we obviously don’t know this from the diagram above. There is no x in the overlap between S and P. Part of that region is shaded out, but the rest could go either way.

What about conditional validity? Can we add an existence assumption that would make this valid? Well, the x we have already drawn lets us know that both S and M exist, so it won’t help to add those premises. What about adding P? That won’t help either. We could add the premise “P exists” but we wouldn’t know whether that P is in the overlap between S and P or in the area to the right, which is just P.

Therefore, this argument is invalid. And when we check the argument against the tables of unconditionally and conditionally valid forms, we see that it is not present.

## VI. Rules and Fallacies

In this section, we are going to identify rules that all valid syllogisms amongst the 256 Aristotelian syllogisms must obey. Seeing these rules will help you understand the structure of this part of logic. We aren’t just assigning the labels “valid” and “invalid” to arguments randomly. Each of the rules we will identify is associated with a fallacy. If you violate the rule, you commit the fallacy.

In the next subsection we are going to outline five basic rules and the fallacies that go with them, along with an addition rule/fallacy pair that can be derived from the initial five. All standard logic textbooks these days use some version of these rules, although they might divide them up differently. Some textbooks also include rules that we have built into our definition of an Aristotelian syllogism in standard form. For instance, other textbooks might have a rule here saying valid syllogisms can’t have four terms, or have to use terms in the same way each time. All of this is built into our definitions of an Aristotelian syllogism and standard form for such a syllogism, so we don’t need to discuss them here.

### Six Rules and Fallacies

#### Rule 1:

The middle term in a valid Aristotelian syllogism must be distributed at least once.

Consider these two arguments:

**Argument 1**

All M are P.

All S are M.

All S are P.

**Argument 2**

All P are M.

All S are M.

All S are P.

Argument 1 is Barbara, and is obviously valid, but if you change it to figure 2, you get Argument 2, which is obviously invalid. What causes this change?

The premises in Argument 2 say that S and P are both parts of M, but they no longer tell us anything about the relationship between S and P. To see why this is the case, we need to bring back a term we saw earlier, distribution. A term is distributed in a statement if the statement makes a claim about every member of that class. So, in “All M are P” the term M is distributed, because the statement tells us something about every single M. They are all also P. The term P is not distributed in this sentence, however. We do not know anything about every single P. We know that M is in P, but not vice versa.

In general, mood-A statements distribute the subject, but not the predicate. This means that when we reverse P and M in the first premise, we create an argument where S and P are distributed, but M is not. This means that the argument is always going to be invalid.

This short argument can show us that arguments with an undistributed middle are always invalid: The conclusion of an Aristotelian syllogism tries to say something about the relationship between S and P. It does this using the relationship those two terms have to the third term M. But if M is never distributed, then S and P can be different, unrelated parts of M. Therefore, arguments with an undistributed middle are invalid. Syllogisms that violate this rule are said to commit the fallacy of the undistributed middle.

#### Rule 2:

If a term is distributed in the conclusion of a valid Aristotelian syllogism, then it must also be distributed in one of the premises.

Suppose instead of changing Barbara from a figure 1 to a figure 2 argument, we changed it to a figure 4 argument. This is what we’d get.

All P are M.

All M are S.

All S are P.

When we changed the argument from figure 1 to figure 2, it ceased to be valid because the middle became undistributed. But this time the middle is distributed in the second premise, and the argument still doesn’t work. You can see this by filling in “animals,” “mammals,” and “dogs,” for S, M, and P.

All dogs are mammals. ⇐ True

All mammals are animals. ⇐ True

All animals are dogs. ⇐ False

This version of the argument has true premises and a false conclusion, so you know the argument form must be invalid. A valid argument form should never be able to take true premises and turn them into a false conclusion. What went wrong here?

The conclusion is a mood-A statement, which means it tries to say something about the entire subject class, namely, that it is completely contained by the predicate class. But that is not what these premises tell us. The premises tell us that the subject class, animals, is actually the broadest class of the three, containing within it the classes of mammals and dogs.

As with the previous rule, the problem here is a matter of distribution. The conclusion has the subject class distributed. It wants to say something about the entire subject class, animals. But the premises do not have “animals” as a distributed class. Premise 1 distributes the class “dogs” and premise 2 distributes the class “mammals.”

Here is another argument that makes a similar mistake:

All M are P.

Some S are not M.

Some S are not P.

This time the conclusion is a mood-O statement, so the predicate term is distributed. We are trying to say something about the entire class P. But again, the premises do not say something about the entire class P. P is undistributed in the major premise.

These examples illustrate rule 2: If a term is distributed in the conclusion, it must also be distributed in the corresponding premise. Arguments that violate this rule are said to commit the fallacy of illicit process. This fallacy has two versions, depending on which term is not distributed. If the subject term is the one that is not distributed, we say that the argument commits the fallacy of an illicit minor. If the predicate term isn’t distributed, we say that the argument commits the fallacy of the illicit major. Some particularly silly arguments commit both.

The justification for this rule is easy enough to see. If the conclusion makes a claim about all members of a class, but the premises only make a claim about some members of the class, the conclusion clearly says more than what the premises justify.

#### Rule 3:

A valid Aristotelian syllogism cannot have two negative premises.

Consider the argument “No P are M, and no M are S, therefore \_\_\_\_\_\_\_\_\_.” Try to find a conclusion about S and P that you can draw from this pair of premises.

Hopefully you have convinced yourself that there is no conclusion to be drawn from the premises above using standard Aristotelian format. No matter what mood you put the conclusion in, it will not follow from the premises. The same thing would be true of any syllogism with two negative premises. We could show this conclusively by running through the 16 possible combinations of negative premises and figures. A more intuitive proof of this rule goes like this: The conclusion of an Aristotelian syllogism must tell us about the relationship between subject and predicate. But if both premises are negative then the middle term must be disjoint, either entirely or partially, from the subject and predicate terms. An argument that breaks this rule is said to commit the fallacy of exclusive premises.

#### Rule 4:

A valid Aristotelian syllogism can have a negative conclusion if and only if it has exactly one negative premise.

Again, let’s start with examples, and try to see what is wrong with them.

**Argument 1:**

All M are P.

All P are M.

Some S are not P.

**Argument 2:**

No P are M.

All S are M.

All S are P.

These arguments are so obviously invalid, you might look at them and say, “Sheesh, is there anything right about them?” Actually, these arguments obey all the rules we have seen so far. Look at Argument 1. Premise 1 ensures that the middle term is distributed. The conclusion is mood O, which means the predicate is distributed, but P is also distributed in the second premise. The argument does not have two negative premises. A similar check will show that Argument 2 also obeys the first three rules.

These arguments illustrate an important premise that is independent of the previous three. You can’t draw a negative conclusion from two affirmative premises, and you cannot drawn an affirmative conclusion if there is a negative premise. Because the previous rule tells us that you can never have two negative premises, we can actually state this rule quite simply: an argument can have a negative conclusion if and only if it has exactly one negative premise. (The phrase “if and only if” means that the rule goes both ways. If you have a negative conclusion, then you must have one negative premise, and if you have one negative premise, you must have a negative conclusion.)

To see why this rule is justified, you need to look at each part of it separately. First, consider the case with the affirmative conclusion. An affirmative conclusion tells us that some or all of S is contained in P. The only way to show this is if some or all of S is in M, and some or all of M is in P. You need a complete chain of inclusion. Therefore, if an argument has a negative premise, it cannot have an affirmative conclusion.

On the other hand, if an argument has a negative conclusion, it is saying that S and P are at least partially separate. But if you have all affirmative premises, you are never separating classes. Also, a valid argument cannot have two negative premises. Therefore, a valid argument with a negative conclusion must have exactly one negative premise.

There is not a succinct name for the fallacy that goes with violating this rule, because this is not a mistake people commonly make. We will call it the negative-affirmative fallacy.

#### Rule 5:

A valid Aristotelian syllogism cannot have two universal premises and a particular conclusion.

This rule is a little different than the previous ones, because it really only applies if you take a Boolean approach to existential import. Consider Barbari, the sometimes maligned step-sister of Barbara:

All M are P.

All S are M.

Some S are P.

This syllogism is not part of the core 15 valid syllogisms we identified with the Venn diagram method using Boolean assumptions about existential import. The premises never assert the existence of something, but the conclusion does. And this is something that is generally true under the Boolean interpretation. Universal statements never have existential import and particular statements always do. Therefore, you cannot derive a particular statement from two universal statements.

Some textbooks act as if the ancient Aristotelians simply overlooked this rule. They say things like “the traditional account paid no attention to the problem of existential import” which is simply false. As we have seen in chapter three, the Latin part of the Aristotelian tradition engaged in an extensive discussion of the issue from the 12th to the 16th centuries, under the heading “supposition of terms.” And at least some people, like William of Ockham, had consistent theories that show why syllogisms like Barbari were valid.

In this textbook, we handle the existential import of universal statements by adding a premise, where appropriate, which makes the existence assumption explicit. So, Barbari should look like this.

All M are P.

All S are M.

Some S exist.\*

Some S are P.

Adding this premise merely gives a separate line in the proof for an idea that Ockham said was already contained in premise 2. And if we make it a practice of adding existential premises to arguments like these, Rule 5 still holds true. You cannot conclude a particular statement from all universal premises. However, in this case, we do have a particular premise, namely, P3. So, if we provide this reasonable accommodation, we can see that syllogisms like Barbari are perfectly good members of the valid syllogism family. We will say, however, that an argument like this that does not provide the extra premise commits the existential fallacy.

### Proving the Rules

For each rule, we have presented an argument that any syllogism that breaks that rule is invalid. It turns out that the reverse is also true. If a syllogism obeys all five of these rules, it must be valid. In other words, these rules are sufficient to characterize validity for Aristotelian syllogisms. It is good practice to actually walk through a proof that these five rules are sufficient for validity. After all, that sort of proof is what formal logic is really all about.

Imagine we have a syllogism that obeys the five rules above. We need to show that it must be valid. There are four possibilities to consider: the conclusion is either mood A, mood E, mood I, or mood O.

If the conclusion is in mood A, then we know that S is distributed in the conclusion. If the syllogism obeys rules 1 and 2, then we know that S and M are distributed in the premises. Rule 4 tells us that both premises must be affirmative, so the premises can’t be I or O. They can’t be E, either, because E does not distribute any terms, and we know that terms are distributed in the premises. Therefore, both premises are in mood A. Furthermore, we know that they are in the first figure, because they have to distribute S and M. Therefore, the syllogism is Barbara, which is valid.

Now suppose the conclusion is in mood E. By rule 4, we have one negative and one affirmative premise. Because mood-E statements distribute both subject and predicate, rules 1 and 2 tell us that all three terms must be distributed in the premises. Therefore, one premise must be E, because it will have to distribute two terms. Since E is negative, the other premise must be affirmative, and since it has to distribute a term, it can’t be I. So we know one premise is A and the other E. If all the terms are distributed, this leaves us four possibilities: EAE-1, EAE-2, AEE-2, and AEE-4. These are the valid syllogisms Celarent, Cesare, Camestres, and Calemes.

Next up, consider the case where the conclusion is in mood I. By rule 4, it has two affirmative premises, and by rule 5 both premises cannot be universal. This means that one premise must be an affirmative particular statement, that is, mood I. But we also know that by rule 1 some premise must distribute the middle term. Since this can’t be the mood-I premise, it must be the other premise, which then must be in mood A. Again we are reduced to four possibilities: AII-1, AII-2, IAI-3, and IAI-4, which are the valid syllogisms Darii, Datisi, Disamis, and Dimatis.

Finally, we need to consider the case where the conclusion is mood O. Rule 4 tells us that one premise must be negative and the other affirmative, and rule 5 tells us that they can’t both be universal. Rules 1 and 2 tell us that M and P are distributed in the premises. This means that the premises can’t both be particular, because then one would be I and one would be O, and only one term could be distributed. So one premise must be negative and the other affirmative, and one premise must be particular and the other universal. In other words, our premises must be a pair that goes across the diagonal of the square of opposition, either an A and an O or an E and an I.

With the AO pair, there are two possibilities that distribute the right terms: OAO-3 and AOO-II. These are the valid syllogisms Bocardo and Baroco. With the EI pair, there are four possibilities, which are all valid. They are the EIO sisters: Ferio, Festino, Ferison, and Fresison.

So, there you have it. Those five rules completely characterize the possible valid Aristotelian syllogisms. Any other patterns you might notice among the valid syllogisms can be derived from these five rules. For instance, Problem (1) in exercise set E of Section II asked if you could have a valid Aristotelian syllogism with two particular premises. If you did that problem, hopefully you saw that the answer was “no.” We could, in fact, make this one of our five rules above. But we don’t need to. When we showed that these five rules were sufficient to characterize validity, we also showed that any other rule characterizing validity that we care to come up with can be derived from the rules we already set out. So, let’s state the idea that a syllogism cannot have two particular premises as a rule, and show how it can be derived. This will be our statement of the rule:

#### Derived Rule 1:

A valid Aristotelian syllogism cannot have two particular premises.

And let’s call the associated fallacy the fallacy of particular premises. To show that this rule can be derived from the previous five, it is sufficient to show that any syllogism that violates this rule will also violate one of the previous five rules. Thus, there will always be a reason, independent of this rule, that can explain why that syllogism is false.

So, suppose we have a syllogism with two particular premises. If we want to avoid violating rule 1, we need to distribute the middle term, which means that both premises cannot be mood I, because mood-I statements don’t distribute any term. We also know that both statements can’t be mood O, because rule 3 says we can’t have two negative premises. Therefore, our syllogism has one premise that is I and one premise that is O. It thus has exactly one negative premise, and by rule 4, must have a negative conclusion, either an E or an O. But an argument with premises I and O can only have one term distributed: if the conclusion is mood O, then two terms are distributed; and if it is mood E then all three terms are distributed. Thus, any syllogism that manages to avoid rules 1, 3, and 4 will fall victim to rule 2. Therefore, any syllogism with two particular premises will violate one of the five basic rules.

1. This chapter is based on *For All X, The Lorain County Remix*, remixed by J. Robert Loftis. [↑](#footnote-ref-1)
2. For most of the arguments in this chapter, we will state the premises first, and the conclusion last, separating the premises from the conclusion with a line. [↑](#footnote-ref-2)