The Logic Book

an open educational resource

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Preface

This textbook is a labor of love, and a true collaborative effort. The four of us have put much effort into creating this book. Most chapters are based on someone else’s open educational resource logic book; some are lightly edited, mainly for formatting and accessibility, but most have been more heavily edited, rewritten, and expanded, to be more in line with how we wanted to teach the material. (Chapter 10 is entirely original to us).

### For logic teachers considering adopting or adapting this book:

For each chapter, I have tried to eliminate any reference to other chapters in the book (“As we learned in Chapter 3 …” for example), as well as internal references by chapter number (so sections are numbered I, II, III, etc., instead of 3.1, 3.2, 3.3). This is so other adopters and adapters of some or all of the material can easily teach the material in any order they wish, or combine it with chapters from other OER textbooks – all you should need to change is where it declares the chapter number at the beginning. One exception is the Introduction, below, which provides an overview of the chapters in this book.

There should be enough flexibility here to choose what kind of course you want to teach; do you want your course to be more deductive-heavy? More inductive? Do you want to teach Venn diagrams, natural deduction, both, neither? The four of us are teaching different portions of the book, and in different orders.

You’ll notice there is no chapter on truth tables. This is for accessibility reasons. Screen readers fail entirely when trying to read symbolic logic formulae, so with translations and natural deduction, I’ve turned the formulae into images and included alt-text that tells the screen reader how to read it. I couldn’t for the life of me figure out how to make truth tables accessible to the visually impaired, so I dropped it. I’ve found that moving from translations and truth conditions directly to natural deduction worked in the classroom much better than I thought it would. Of course, there are plenty of OER books that have chapters on truth tables if you’d like to insert that material.

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### Gratitude:

We wish to heartily thank the authors of the OER books on which we’ve based these chapters; a full list is below. Many thanks go to the Clayton State University critical thinking students of Spring Semester, 2025 (all 323 of you!), for helping us pilot this textbook. We appreciate your patience, and your attention to detail pointing out inconsistencies, typos, and falsehoods. And we extend our deepest gratitude to Affordable Learning Georgia for the generous grant that made this work possible.

### Source material:

*For All X: The Lorain County Remix*, remixed by J. Robert Loftis (2005-2017). This book’s webpage is here (link opens in a new window): [For All X](https://forallxremix.org/). (Chapters 1, 2, 3, 4)

*Fundamental Methods of Logic*, by Matthew Knachel (2017). The book can be accessed here (link opens in a new window): [Fundamental Methods](https://open.umn.edu/opentextbooks/textbooks/fundamental-methods-of-logic) (Chapters 5, 7, 9, 11)

*Introduction to Logic and Critical Thinking*, by Matthew J. Van Cleave (2016). This book can be accessed here (link opens in a new window): [Logic and Critical Thinking](https://open.umn.edu/opentextbooks/textbooks/introduction-to-logic-and-critical-thinking). (Chapter 6)

*Logical Reasoning*, by Bradley H. Dowden (2011-2017).This book can be accessed here (link opens in a new window): [Logical Reasoning](https://open.umn.edu/opentextbooks/textbooks/745). (Chapter 8)

# Introduction

Congratulations, you are enrolled in a course that has been taught for nearly twenty-five hundred years! We study how to reason correctly: when our evidence justifies our conclusions and how to detect liars and cheats. The curriculum dates to philosophers such as Aristotle, who worked in the Greek city of Athens. Traditionally, Critical Thinking is thought of as the foundation of a well-rounded education.

Critical Thinking used to be called ‘logic,’ but owing to twentieth-century developments, the field grew increasingly technical, and it now makes more sense to teach classical logic as Critical Thinking. Critical Thinking is described as the ‘Art of Arts’ because no matter what you do with your life, Critical Thinking helps you to do it better. Physics, law, customer service, mixed martial arts, theatre, parenting, take your pick – knowing how to think through issues, support conclusions with evidence, and call out bad reasoning is vital, especially in our toxic, social-media landscape that is littered with lies and bad arguments. Chapters one and two offer an overview of Critical Thinking with attention to when conclusions are guaranteed or merely probable. Chapters three, four, five, and six show how to clarify and evaluate what is really going on when someone asks you to believe something based on the reasons that they give. In chapter seven, we study common tricks (fallacies) designed to make bad arguments look good. Chapters eight, nine, ten, and eleven pursue these themes as they play out in scientific and ordinary (day-to-day) decision making.

Some closing notes: Aristotle is often credited with creating logic. At best, this is only partially true, and only as regards the particular tradition that we’ll study. Aristotle worked out of the Academy, an ancient thinktank of sorts, founded in the fourth-century BCE by the philosopher Plato (and named after the Greek Olympic victor Academus). We owe to this school developments in fields such as geometry, astronomy, physics and politics. Aristotle did not work alone. In particular, Academicians (those working at the Academy) focused on mathematics. It is likely that the Greek mathematical tradition borrowed from ancient Egypt and Mesopotamia. Around the sixth-century BCE, Greek thinkers grow increasingly interested in mathematics as an instance of ‘best-case’ reasoning, applying it to fields such as physics (they gave us the term ‘Atom’ for the smallest, indestructible particle of matter). By way of example, consider the Pythagorean theorem that many of us had to memorize in high school (fun fact, Pythagoras, who lived in the sixth-century BCE, didn’t create the theorem; we find it in Babylonian and Indian works that predate him by centuries). The point is, mathematics is generally presented as a way to arrive at certainty. As Greek thinkers worked to employ mathematical-type reasoning better to understand the world, it is unsurprising that Aristotle and others sought to incorporate mathematical precision into argumentation, i.e., the process of discovering things by thinking carefully about our evidence. Consider this example developed from Aristotle’s work:

Socrates is human.

Humans are mortal.

Therefore, Socrates is mortal.

If the premises are true, the conclusion is guaranteed. Critical Thinking discovered lots of methods to arrive at necessary and probable conclusions. Add in ways to figure out when an argument is bad and how to call out people trying to fool you (or to discover when honestly intended reasoning is simply mistaken) and you have a powerful set of tools called the ‘Art of Arts’ for good reason.

Finally, we should not think that logic is a Western invention. First of all, we have lost lots of writings that predate the Academy, and it would be silly to think that no one except the Greeks ever thought about this stuff. Secondly, and more to the point, we find writings on logic in the ancient Chinese Mohist canon (dating to the fifth-century BCE) and Indian, fourth-century BCE Nyaya, Vaisheshika and Nagarjuna thought. So, the Greeks didn’t create logic. Their contributions and those of thinkers in the Greek tradition are numerous and important, but they weren’t the only civilization reasoning along these lines.

# Chapter 1: What Is Logic?[[1]](#footnote-1)

## I. Arguments, Premises, Conclusion

Logic is a part of the study of human reason, the ability we have to think abstractly, solve problems, explain the things that we know, and infer new knowledge on the basis of evidence. Traditionally, logic has focused on the last of these items, the ability to make inferences on the basis of evidence. This is an activity you engage in every day. In logic, we don’t use the word “argument” just to refer to two people disagreeing. We use “argument” to refer to the attempt to show that certain evidence supports a conclusion.

A logical argument is structured to give someone a reason to believe some conclusion. Here is an argument about a game of Clue written out in a way that shows its structure.

In a game of Clue, the possible murder weapons are the knife, the candlestick, the revolver, the rope, the lead pipe, and the wrench.

The murder weapon was not the knife.

The murder weapon was also not the revolver, the rope, the lead pipe, or the wrench.

Therefore, the murder weapon was the candlestick.

In the argument above, the first three statements are the evidence. We call these the **premises**. The word “therefore” indicates that the final statement (written below the line) is the **conclusion** of the argument. If you believe the premises, then the argument provides you with a reason to believe the conclusion.

We can define **logic** then more precisely as the part of the study of reasoning that focuses on argument. In more casual situations, we will follow ordinary practice and use the word “logic” to either refer to the business of studying human reason or the thing being studied, that is, human reasoning itself. While logic focuses on argument, other disciplines, like decision theory and cognitive science, deal with other aspects of human reasoning, like abstract thinking and problem solving more generally.

Logic, as the study of argument, has been pursued for thousands of years by people from civilizations all over the globe. The initial motivation for studying logic is generally practical. Given that we use arguments and make inferences all the time, it only makes sense that we would want to learn to do these things better. Once people begin to study logic, however, they quickly realize that it is a fascinating topic in its own right. Thus the study of logic quickly moves from being a practical business to a theoretical endeavor people pursue for its own sake.

In order to study reasoning, we have to apply our ability to reason to our reason itself. This reasoning about reasoning is called **metareasoning**. It is part of a more general set of processes called **metacognition**, which is just any kind of thinking about thinking. When we are pursuing logic as a practical discipline, one important part of metacognition will be awareness of your own thinking, especially its weakness and biases, as it is occurring. More theoretical metacognition will be about attempting to understand the structure of thought itself.

Whether we are pursuing logic for practical or theoretical reasons, our focus is on argument. The key to studying argument is to set aside the subject being argued about and to *focus on the way it is argued for*. The above example was about a game of Clue. However, the kind of reasoning used in that example was just the process of elimination. The process of elimination can be applied to any subject. Suppose a group of friends is deciding which restaurant to eat at, and there are six restaurants in town. If you could rule out five of the possibilities, you would use an argument just like the one above to decide where to eat.

Because logic sets aside what an argument is about, and just looks at how it works rationally, logic is said to have ***content neutrality***. If we say an argument is good, then the same kind of argument applied to a different topic will also be good. If we say an argument is good for solving murders, we will also say that the same kind of argument is good for deciding where to eat, what kind of disease is destroying your crops, or who to vote for.

When logic is studied for theoretical reasons, it typically is pursued as formal logic. In formal logic we get content neutrality by replacing parts of the argument we are studying with abstract symbols. For instance, we could turn the argument above into a formal argument like this:

There are six possibilities: A, B, C, D, E, and F.

A is false.

B, D, E, and F are also false.

Therefore, the correct answer is C.

Here we have replaced the concrete possibilities in the first argument with abstract letters that could stand for anything. This lets us see the ***formal structure*** of the argument, which is why it works in any domain you can think of. In fact, we can think of formal logic as the method for studying argument that uses abstract notation to identify the formal structure of argument.

Formal logic is closely allied with mathematics, and studying formal logic often has the sort of puzzle-solving character one associates with mathematics. You will see this when we get to the chapters on formal logic.

When logic is studied for practical reasons, it is typically called **critical thinking**. We will define critical thinking narrowly as **the use of metareasoning to improve our reasoning in practical situations.** Sometimes we will use the term “critical thinking” more broadly to refer to the results of this effort at self-improvement. You are “thinking critically” when you reason in a way that has been sharpened by reflection and metareasoning. A critical thinker is someone who has both sharpened their reasoning abilities using metareasoning, and deploys those sharpened abilities in real world situations.

Critical thinking is generally pursued as informal logic, rather than formal logic. This means that we will keep arguments in ordinary language and draw extensively on your knowledge of the world to evaluate them. In contrast to the clarity and rigor of formal logic, informal logic is suffused with ambiguity and vagueness. There are problems with multiple correct answers, and problems where reasonable people can disagree with what the correct answer is. This is because you will be dealing with reasoning in the real world, which is messy.

Our main goal in studying arguments is to separate the good ones from the bad ones. The argument about Clue we saw earlier is a good one, based on the process of elimination. It is good because, if I’ve got all the premises right, the conclusion will also be right.

### Statements

So far we have defined logic as the study of argument and outlined its relationship to related fields. To go any further, we are going to need a more precise definition of what exactly an argument is. We have said that an argument is not simply two people disagreeing; it is an attempt to prove something using evidence. More specifically**,** anargument is composed of statements intended to support a conclusion. In logic, we define a **statement** as *a unit of language (typically a sentence) that can be true or false*. The idea that statement is a sentence that can be true or false is also captured by saying that every statement has a *truth value*. “All cats are dogs,” is a sentence whose truth value is *false*. “All cats are animals,” is a sentence whose truth value is *true.* All of the items below are statements.

(a) Tyrannosaurus rex went extinct 65 million years ago.

(b) Tyrannosaurus rex went extinct last week.

(c) On this exact spot, 100 million years ago, a T. rex laid a clutch of eggs.

(d) George W. Bush is the king of Jupiter.

(e) Murder is wrong.

(f) Abortion is murder.

(g) Abortion is a woman’s right.

(h) Lady Gaga is pretty.

(i) Murder is the unjustified killing of a person.

(j) The slithy toves did gyre and gimble in the wabe.

(k) The murder of logician Richard Montague was never solved.

Because a statement is something that has a truth value, statements include truths like (a) and falsehoods like (b). A statement can also be something that that must either be true or false, but we don’t know which, like (c). A statement can be something that is completely silly, like (d). Statements in logic include statements about morality, like (e), and things that in other contexts might be called “opinions,” like (f) and (g). People disagree strongly about whether (f) or (g) are true, but it is definitely possible for one of them to be true. The same is true about (h), although it is a less important issue than (f) and (g). A statement in logic can also simply give a definition, like (i). This sort of statement announces that we plan to use words a certain way, which is different from statements that describe the world, like (a), or statements about morality, like (f). Statements can include nonsense words like (j), because we don’t really need to know what the statement is about to see that it is the sort of thing that can be true or false. All of this relates back to the content neutrality of logic. The statements we study can be about dinosaurs, abortion, Lady Gaga, and even the history of logic itself, as in statement (k), which is true.

We are treating statements primarily as units of language or strings of symbols, and most of the time the statements you will be working with will just be words printed on a page. However, it is important to remember that statements are also what philosophers call “speech acts.” They are actions people take when they speak (or write). If someone makes a statement, they are typically telling other people that they believe the statement to be true, and will back it up with evidence if asked to. When people make statements, they always do it in a context—they make statements at a place and a time with an audience. Often the context in which statements are made will be important for us, so when we give examples, statements, or arguments, we will sometimes include a description of the context. When we do that, we will give the context in italics. For example:

The Earth is 4.5 billion years old.

*Susan is arguing with a young-earth creationist.* The Earth is 4.5 billion years old.

*From a college-level textbook.* The Earth is 4.5 billion years old.

In each of these, “The Earth is 4.5 billion years old” is the statement. The sentences in italics are our inclusion of the context in which each statement was made.

The word ‘statements’ in this text does not include questions, commands, exclamations, or sentence fragments. Someone who asks a question like “Does the grass need to be mowed?” is typically not claiming that anything is true or false. Generally, questions will not count as statements, but answers will. “What is this course about?” is not a statement. “No one knows what this course is about,” is a statement.

For the same reason commands do not count as statements for us. If someone bellows “Mow the grass, now!” they are not saying whether the grass has been mowed or not. You might infer that they believe the lawn has not been mowed, but then again maybe they think the lawn is fine and just want to see you exercise.

An exclamation like “Ouch!” is also neither true nor false. On its own, it is not a statement. We will treat “Ouch, I hurt my toe!” as meaning the same thing as “I hurt my toe.” The “ouch” does not add anything that could be true or false.

Finally, a lot of possible strings of words will fail to qualify as statements simply because they don’t form a complete sentence. In your composition classes, these were probably referred to as sentence fragments. This includes strings of words that are parts of sentences, such as noun phrases like “The tall man with the hat” and verb phrases, like “ran down the hall.” Phrases like these are missing something they need to make a claim about the world. The class of sentence fragments also includes completely random combinations of words, like “The up if blender route,” which doesn’t even have the form of a statement about the world.

When we study argument, we need to express things as statements, because arguments are composed of statements. Thus, if we encounter a rhetorical question while examining an argument, we need to convert it into a statement. “Don’t you think the lawn needs to be mowed?” will become “The lawn needs to be mowed.” Similarly, commands will become ‘should’ statements. “Mow the lawn, now!” will need to be transformed into “You should mow the lawn.”

The latter kind of change will be important in critical thinking, because critical thinking often studies arguments whose goal is to an get audience to do something. These are called practical arguments. Most advertising and political speech consists of practical arguments, and these are crucial topics for critical thinking.

### Arguments

Once we have a collection of statements, we can use them to build arguments. **An argument is a connected series of statements, one or more of which is designed to provide support for another statement.** Let’s start with an example of an argument given to an external audience. This passage is from an essay by Peter Singer called “Famine, Affluence, and Morality” in which he tries to convince people in rich nations that they need to do more to help people in poor nations who are experiencing famine.

*A contemporary philosopher writing in an academic journal*. If it is in our power to prevent something bad from happening, without thereby sacrificing anything of comparable moral importance, we ought, morally, to do so. Famine is something bad, and it can be prevented without sacrificing anything of comparable moral importance. So, we ought to prevent famine.[[2]](#footnote-2)

Singer wants his readers to work to prevent famine. This is represented by the last statement of the passage, “we ought to prevent famine,” which is called the conclusion of the passage. The **conclusion** of an argument is the statement that the argument is trying to convince the audience of. The statements that do the convincing are called the **premises**. In this case, the argument has three premises: (1) “If it is in our power to prevent something bad from happening, without thereby sacrificing anything of comparable moral importance, we ought, morally, to do so,” (2) “Famine is something bad,” and (3) “It can be prevented without sacrificing anything of comparable moral importance.”

Now let’s look at an example of internal reasoning.

*Jack arrives at the track, in bad weather.* There is no one here. I guess the race is not happening.

In the passage above, the words in italics explain the context for the reasoning, and the words in regular type represent what Jack is actually thinking to himself. This passage again has a premise and a conclusion. The premise is that no one is at the track, and the conclusion is that the race was canceled. The context gives another reason why Jack might believe the race has been canceled: the weather is bad. You could view this as another premise—it is very likely a reason Jack has come to believe that the race is canceled. In general, when you are looking at people’s internal reasoning, it is often hard to determine what is actually working as a premise and what is just working in the background of their unconscious.

When people give arguments to each other, they typically use words like “therefore” and “because.” These are meant to signal to the audience that what is coming is either a premise or a conclusion in an argument. Words and phrases like “because” signal that a premise is coming, so we call these premise indicators. Similarly, words and phrases like “therefore” signal a conclusion and are called conclusion indicators. The argument from Peter Singer uses the conclusion indicator word, “so.” Here is an incomplete list of indicator words and phrases in English.

**Premise Indicators:**

because, as, for, since, given that, for the reason that, may be inferred from, owing to, is evidenced by

**Conclusion indicators:**

therefore, thus, hence, so, consequently, it follows that, in conclusion, as a result, it must be the case, accordingly, this implies that, this entails that, we may infer that

The two passages we have looked at in this section so far have been simply presented as quotations. But often it is extremely useful to rewrite arguments in a way that makes their logical structure clear. One way to do this is to use something called “canonical form.” An argument written in canonical form has each premise numbered and written on a separate line. Indicator words and other unnecessary material should be removed from the premises. Although you can shorten the premises and conclusion, you need to be sure to keep them all complete sentences with the same meaning, so that they can be true or false. The argument from Peter Singer, above, looks like this in canonical form:

If we can stop something bad from happening, without sacrificing anything of comparable moral importance, we ought to do so.

Famine is something bad.

Famine can be prevented without sacrificing anything of comparable moral importance.

We ought to prevent famine.[[3]](#footnote-3)

Each premise has been written on its own line; the conclusion is written last below the line. The statements have been paraphrased slightly for brevity, and the indicator word “so” has been removed. Also notice that the “it” in the third premise has been replaced by the word “famine,” so that statements reads naturally on its own.

Similarly, we can rewrite the argument Jack gives at the racetrack, like this:

There is no one at the race track.

The race is not happening.

Notice that we did not include anything from the part of the passage in italics. The italics represent the context, not the argument itself. Also, notice that the “I guess” has been removed. When we write things out in canonical form, we write the content of the statements, and ignore information about the speaker’s mental state, like “I believe” or “I guess.”

One of the first things you have to learn to do in logic is to identify arguments and rewrite them in canonical form. This is a foundational skill for everything else we will be doing in this text, so we are going to go over an example here, and there will be more in the exercises. The passage below is paraphrased from the ancient Greek philosopher Aristotle.

*An ancient philosopher, writing for his students*. Again, our observations of the stars make it evident that the earth is round. For quite a small change of position to south or north causes a manifest alteration in the stars which are overhead.[[4]](#footnote-4)

The first thing we need to do to put this argument in canonical form is to identify the conclusion. The indicator words are frequently the best way to do this. The phrase “make it evident that” is a conclusion indicator phrase. He is saying that everything else is evidence for what follows. So we know that the conclusion is that the earth is round. “For” is a premise indicator word—it is sort of a weaker version of “because.” Thus the premise is that the stars in the sky change if you move north or south. In canonical form, Aristotle’s argument that the earth is round looks like this.

There are different stars overhead in the northern and southern parts of the earth.

The earth is spherical in shape.

*The ultimate test of whether something is an argument* is simply whether some of the statements provide reason to believe another one of the statements. If some statements support others, you are looking at an argument. The speakers in these two cases use indicator phrases to let you know they are trying to give an argument.

A final bit of terminology for this section. An **inference** is the act of coming to believe a conclusion on the basis of some set of premises. When Jack in the example above saw that no one was at the track, and came to believe that the race was not on, he was making an inference. We also use the term inference to refer to the connection between the premises and the conclusion of an argument. If your mind moves from premises to conclusion, you make an inference, and the premises and the conclusion are said to be linked by an inference. In that way inferences are like argument glue: they hold the premises and conclusion together.

## II. Inductive Arguments, Deductive Arguments

Since the time of Aristotle, the study of logic has been divided into the study of two distinct types of reasoning—deductive and inductive. One way to think of the difference between these types of arguments is to think of deductive arguments in terms of “necessity,” and inductive arguments in terms of probability. In other words, we distinguish deductive arguments from inductive arguments in terms of the strength of the inferential link between premises and conclusion. The strongest inferential link possible would be one where the premises make the conclusion follow from the premises necessarily—i.e., where the premises *force* us to draw the conclusion. An argument like would be a **deductive argument.** By contrast, an argument whose purpose is to make its conclusion “probable” is an **inductive argument**. Here are examples of each type:

**Deductive**

All cats are animals.

Rob Halford is a cat.

Therefore, Rob Halford is an animal.

Given the premises of this argument, we are forced to draw the conclusion that Rob Halford is an animal. Since the link between premises and conclusion here is one of necessity, and we are forced to draw this conclusion, this is a deductive argument.

**Inductive**

Ninety-nine percent of the people who like baseball live to be 120 years old. I like baseball, so I’ll probably live to be 120 years old.

Given the premises of this argument, we aren’t *forced* to draw the conclusion, but the premises do make the conclusion highly probable.

At this point we can formulate brief, useful definitions for deductive and inductive arguments:

**Deductive Argument**: An argument in which the conclusion is claimed to follow from the premises with strict necessity, i.e., given the premises, it is claimed that we must draw the conclusion.

**Inductive argument**: An argument in which the conclusion is claimed to follow from the premises with probability, i.e., given the premises, it is claimed that the conclusion is probable.

### Distinguishing Deductive and Inductive Arguments

In trying to distinguish between deductive and inductive arguments, the best thing to focus on is the relationship between the premises and the conclusion. It may be helpful to think in terms of two different kinds of light switches. Deductive arguments operate on something like a light switch that's “off” or “on” with no in-between—there’s light, or there isn’t. For deductive arguments the conclusion either follows from the premises, or it doesn’t. There are no *varying degrees* of validity. Inductive arguments operate on something like a dimmer switch: just as a dimmer switch gives varying degrees of light intensity, inductive arguments have varying degrees of strength.



Inductive Argument: Dimmer Switch on the left

Deductive Argument: On/Off switch on the right

#### Deductive Arguments

**Deductive arguments** are always a matter of necessity. For example:

“The day after Monday is always Friday. Today is Monday, so it follows necessarily that tomorrow is Friday.”

The first premise is false, but if it *were* true, the conclusion follows necessarily—it’s guaranteed by the premises. It’s like “two plus two equals four.” If I tell you I have two apples in one hand and two apples in the other, then you have to conclude that I’ve got four apples. Nothing else would fit with those premises, just as nothing else would fit after the “equals” in the addition problem. Here’s another example: If I tell you that all dogs are animals and that Henri is a dog, then you have no choice but to conclude that Henri is an animal. The premises make that conclusion necessary. And if I say that all dolphins are Martians, and Henri is a dolphin, the only conclusion that you can draw is that Henri is a Martian. In all these cases the light switch is moved to the “on” position.

But if I tell you that all dogs are animals, and that Henri is a cat, it DOES NOT follow that Henri is an animal. In that case the light stays off. Now, you and I both know that cats are animals, so that if Henri is a cat then he must be an animal, but the premises don’t say that, and we can’t assume it if it’s not in the premises—so this argument doesn’t guarantee/necessitate that conclusion. In other words, the premises don’t *force* us to draw that conclusion.

The absolutely central thing to recognize about deductive arguments is that they have a “form” or structure, and it’s that form that makes it deductive, and it’s that form alone, and nothing else, that determines whether the argument is valid or invalid. Now, the handy thing about that is that deductive arguments can all be represented by a diagram, or a timeline, or a drawing, or by reducing them to their skeletal form by substituting letters for the terms/statements in the original argument. If you can diagram it / reduce it to a form, and can tell just from the form whether it’s good or bad, then you know it's a deductive argument. You can't do that with inductive arguments — you have to know the content to tell whether it's good or bad.

Here are two examples:

All dogs are animals.

Henri is a dog.

Therefore, Henri is an animal.

The form of this argument is:

All D are A

H is D

Therefore, H is A.

“H is A” is guaranteed by the premises here. Nothing else will go in its place. This argument, then, or rather its form, is what we will call “valid.” Any argument in which the terms are arranged that way will be a valid argument. (The concept of validity will be discussed in more detail below). Here’s another deductive argument:

Steve is taller than Cheryl, and Cheryl is taller than Mary. It follows that Steve is taller than Mary.

You can write this out using different sized letters to represent the people, and use the letters S, C and M. You’d get something like this:

Three letters in a row, S, C, and M. S is much larger than C, and C is larger than M.

Since you can represent the argument like this, it’s deductive.

#### Inductive Arguments

**Inductive arguments** are always a matter of probability. The premises of an inductive argument can only make a conclusion more or less probable, but never guaranteed or necessary. Consider the following argument:

People who get flu shots have a 93% chance of not getting the flu. I just got a flu shot, therefore I probably won't get the flu.

The conclusion here is not guaranteed, of course. Some people who get a flu shot obviously DO get the flu (7% to be precise). But 93% is a high degree of probability (anything over 50% is considered probable), so the premises of this argument make the conclusion highly probable. An inductive argument where the premises make the conclusion probable is called “strong.” (Like validity, the concept of strength will be discussed in more detail below).

An essential point here, and one that bears frequent repeating: It will strike you as completely counter-intuitive at first, but in trying to determine whether deductive and inductive arguments support their conclusions, it doesn’t matter whether any of the statements in the arguments are true. Taking the flu shot example—I have no actual idea what the numbers are on flu shots, and I didn’t just get one. But logic doesn’t care about that. Logic is only concerned with whether, given what the premises say, the premises support the conclusion. You are going to have to continually tell yourself not to answer questions about what kinds of arguments you are looking at, and whether their premises support their conclusions, based on your knowledge of whether their conclusions are true. Here’s an inductive argument example to illustrate:

NASA astronauts have gone to the Moon 647,832 times so far this year. Every single time they’ve landed on the moon a little green man comes out of a hole in the ground and serves them hamburgers and beer. Therefore, the next time astronauts make a trip to the moon, that little green man will probably come out and serve them hamburgers and beer.

The conclusion of this argument follows with an exceptionally high degree of probability. And none of it is true. But in logic we don’t care about that when we are trying to determine whether the premises of an argument support its conclusion.

Another helpful way of recognizing deductive and inductive arguments is to familiarize ourselves with a handful of common types of arguments that fall under each heading. If you recognize that something is an argument based on mathematics, for example, you know that that argument is deductive. If you recognize that something is a causal inference, by contrast, you know that that argument is inductive.

### Common Types of Deductive Arguments

In deductive arguments like the ones in this section, the conclusion is supposed to contain only information that is already in the premises. You simply shuffle information around. (The word “deductive” comes from the Latin words *de* and *ducere*, and means “to take out”).

An **argument based on mathematics** is one in which the arguer draws a conclusion by doing some mathematical computation: addition, subtraction, division, multiplication, etc. Imagine a person who goes to the hardware and puts four hammers and six screwdrivers in their cart, and that each item costs two dollars with tax. They conclude that their purchase will amount to twenty dollars. Arguments that depend on calculations like this are always best seen as deductive. (Be advised, though, that most statistical reasoning, though it involves math, is almost always inductive).

An **argument from definition** is one in which the conclusion of an argument involves stating the definition of a word in the premise. For example, you could argue that since Humbertimus is riding a tricycle, it follows that he is riding something that has three wheels. Or someone could argue that someone is duplicitous because they are always being deceitful. Arguments from definition are always deductive.

There are three types of **syllogism** (a syllogism is an argument with two premises and one conclusion):

**Categorical Syllogism**: A syllogism in which each statement begins with the word “All,” “No,” or “Some.” For example:

All Mustangs are Fords

Some Mustangs are red sports cars.

Therefore, some Fords are red sports cars.

**Hypothetical Syllogism**: A syllogism containing at least one premise which is a conditional (if…then…) statement. For example:

If Atlanta is the capital of Georgia, then Atlanta is a southern city.

Atlanta is the capital of Georgia.

Therefore, Atlanta is a southern city.

**Disjunctive Syllogism**: A syllogism in which one of the premises is an either/or statement:

Either George Bush was a president, or Beyoncé was a president.

George Bush was not a president.

Therefore, Beyoncé was a president.

### Common Types of Inductive Arguments

In inductive arguments, as in the examples below, the conclusion goes beyond the premises and adds something to the information given. You try to give as much evidence as you can for the conclusion. (“Inductive” also comes from Latin: *in* plus *ducere* means “to put in”).

A **prediction** is an argument that uses information about the past to make a claim about what will happen in the future. For example, someone might argue that since every squirrel people have ever seen has eaten nuts, that probably the next squirrel someone sees will also be a nut eater. Or someone might argue that, given all the data we have linking smoking with lung disease, that a person who smokes three packs of cigarettes ever day will likely develop lung disease. Predictions never deal with absolutes, but only with probabilities, so these types of arguments are almost always best read as inductive.

A **causal inference** is an argument that proceeds from knowledge of a cause and makes a claim about a subsequent effect, or proceeds from knowledge of an effect and makes a claim about a previous cause. For example, you might argue that since Kat hit their hand with a hammer, they developed a bruise. Or you might argue that other way, that since Kat is sniffling and sneezing and coughing, they must have caught a cold. (\*Note: it’s important to distinguish between predictions and causal inferences; predictions always gather data from the past and try to establish what will happen in the future. Causal inferences where you know the effect and are reasoning out what caused it always draw a conclusion about something that has *already happened.* Causal inferences where you know the cause and are reasoning out the effect could have a conclusion about the past, present, or future, whenever we think the effect would most likely happen).

An **argument from authority** is one in which the arguer claims that something must be the case because someone taken by the arguer to be an expert has said so. Arguments from authority then typically look like this. “X said Y must be the case. Since X is an expert, Y must be the case.” For example: “The cable repair person said there must be something wrong my cable box. Since they’re the expert, it’s probably true that there’s something wrong with my cable box.” Experts can be wrong, though, and they sometimes fail to tell the truth, so these types or arguments are always only a matter of probability.

An **argument based on signs** is one in which the arguer claims that some conclusion follows simply because of something stated on a sign. Here we have to interpret the word “sign” broadly, to include literal signs like stop-signs and theater marquees, but also any visual messages produced by an intelligent being: things like clocks, pop-up messages, product labels, price-tags, a row of orange cones in the road. Any visual clue someone left as a form of communication. Signs can be mistaken, though, or misplaced, or out of date, so these arguments never make their conclusions strictly necessary (they don’t *force* us to draw their conclusions), but only probable.

A **generalization** is an argument that proceeds from knowledge of specific cases to a general claim about an entire group, or *all* members of a certain class of things. For example, you might argue that since the people from Wisconsin you met on your last vacation were really nice, that *all* people from Wisconsin are really nice. Or you might claim that all Waffle Houses make great hashbrowns, since every Waffle House you’ve eaten at has always had great hashbrowns.

An **argument from analogy** is an inductive argument in which the arguer compares things and draws a conclusion based on the similarities between those things. We make these kinds of arguments whenever we ask people we trust about their experiences with things we are interested in pursuing. Say, for example, you are thinking of buying the latest iPhone. You might ask your friend who has the latest model whether they are satisfied with the phone. If they’re very satisfied, you might conclude that you will also be very satisfied if you get the same (exactly similar) phone. These types of arguments at best make their conclusions probable.

### Summing Up

The key to distinguishing Inductive and Deductive arguments is to focus on the support relationship between the premises and conclusion.

An argument is best interpreted as deductive if:

1. The support relationship between premises is one of *necessity*.
2. You can isolate the form of the argument by replacing terms with letters, drawing a diagram or circles or a timeline, etc., and you can evaluate the argument as good or bad based on the form.
3. It has a recognizable deductive form.

An argument is best interpreted as inductive if:

1. The support relationship between premises and conclusion is one of *probability*.
2. Trying to construct a diagram or abstract representation of the argument does not help you evaluate it.
3. It shares recognizable features with common inductive argument types.

## III. Evaluating Arguments

### Two Ways an Argument Can Go Wrong

Arguments are supposed to lead us to the truth, but they don’t always succeed. There are two ways they can fail in their mission. First, they can simply start out wrong, using false premises. Consider the following argument:

It is raining heavily.

If you do not take an umbrella, you will get soaked.

You should take an umbrella.

If premise (1) is false—if it is sunny outside—then the argument gives you no reason to carry an umbrella. The argument has failed its job. Premise (2) could also be false: Even if it is raining outside, you might not need an umbrella. You might wear a rain poncho or keep to covered walkways and still avoid getting soaked. Again, the argument fails because a premise is false.

Even if an argument has all true premises, there is still a second way it can fail. Suppose for a moment that both the premises in the argument above are true. It is actually raining heavily. You do not own a rain poncho. You need to go places where there are no covered walkways. Now does the argument show you that you should take an umbrella? Not necessarily. Perhaps you enjoy walking in the rain, and you would like to get soaked. In that case, even though the premises were true, the conclusion would be false. The premises, although true, do not support the conclusion. Back when we defined an inference, we said it was like argument glue: it holds the premises and conclusion together. When an argument goes wrong because the premises do not support the conclusion, we say there is something wrong with the inference.

Consider another example:

You are reading this book.

This is a logic book.

You are a logic student.

This is not a terrible argument. Most people who read this book are logic students. Yet, it is possible for someone besides a logic student to read this book. If your roommate picked up the book and thumbed through it, they would not immediately become a logic student. So the premises of this argument, even though they are true, do not guarantee the truth of the conclusion. Its inference is less than perfect.

Again, for any argument, there are two ways that it could fail. First, one or more of the premises might be false. Second, the premises might fail to support the conclusion. Even if the premises were true, the form of the argument might be weak, meaning the inference is bad.

In logic, we are almost exclusively concerned with evaluating the quality of inferences, not the truth of the premises. The truth of various premises will be a matter of whatever specific topic we are arguing about, and, as we have said, ***logic is content neutral***.

Remember that whether the things in the arguments are true is irrelevant to determining what kind of argument it is (deductive/inductive) and this goes as well for determining whether it’s valid/invalid or strong/weak. **Logic is largely unconcerned with whether the statements in the argument are true.** Instead, logic is almost exclusively concerned with whether the premises of an argument, true or false, provide *support* for the conclusion, true or false. Here are two examples, one inductive and one deductive, where everything in the arguments is false, and yet the premises support the conclusion:

Here's the inductive example:

The average temperature for December in Atlanta has been 142 degrees every year for last 113 years. Probably next year the average temperature for December in Atlanta will be 142 degrees.

Given the premise here (which is false), the conclusion follows with a high degree of probability.

Here’s the deductive example:

All cats are Martians.

Barack Obama is a cat.

Therefore Barack Obama is a Martian.

Given what the premises here say (and they are false as a matter of fact) we MUST draw the conclusion here. The premises make that conclusion *necessary*.

### Deductive Arguments

Deductive and inductive arguments are evaluated differently, so we have different vocabulary for whether each type of logic succeeds or fails. A deductive argument with good logic is called valid; a deductive argument with bad logic is called invalid.

#### Validity

For the purposes of this textbook, we will adopt the following definitions for validity:

**Valid**: The conclusion is supported by (follows from) the premises. So: *if* the premises of a valid argument are true, it is impossible for the conclusion to be false.

**Invalid**: The conclusion is *not* supported by (does *not* follow from) the premises. So: *if* the premises of an invalid argument are true, it is still possible for the conclusion to be false.

Another way to put this is to say that the only thing that matters for validity is whether the premises, as stated, support the conclusion. So, for example, you can have valid and invalid arguments with all true premises and a true conclusion, valid and invalid arguments with false premises and a true conclusion, valid and invalid arguments with one false and one true premise and a true conclusion, etc. The only thing you cannot have with a valid argument is all true premises and a false conclusion.

A **valid** deductive argument is one where the premises support the conclusion. Given that, it is also not possible for a valid deductive argument to have true premises and a false conclusion. In other words, given a valid argument (the premises support the conclusion), then *if* the premises are true, the conclusion must also be true. **But**, **the premises of a valid argument *do not have to be true***. There are valid arguments with false premises for example, in which case the argument can have either true or false premises. So, the only arrangement of truth values that is not possible is a valid argument with true premises and false conclusion.

It’s important to always keep in mind that valid arguments can have false conclusions, because people naturally tend to think that any argument must be good if they agree with the conclusion. And the more passionately people believe in the conclusion, the more likely we are to think that any argument for it must be brilliant. Conversely, if the conclusion is something we don’t believe in, we naturally tend to think the argument is poor. And the more we don’t like the conclusion, the less likely we are to like the argument.

But this is not the correct way to evaluate inferences at all. The quality of the inference is entirely independent of the truth of the conclusion. You can have great arguments for false conclusions and horrible arguments for true conclusions. We have trouble seeing this because of biases built deep in the way we think called “cognitive biases.” A cognitive bias is a habit of reasoning that can be dysfunctional in certain circumstances. Generally, these biases developed for a reason, so they serve us well in many or most circumstances. But cognitive biases also systematically distort our reasoning in other circumstances, so we must be on guard against them.

There is a particular cognitive bias that makes it hard for us to recognize when a poor argument is being given for a conclusion we agree with. It is called “confirmation bias” and it is in many ways the mother of all cognitive biases. Confirmation bias is the tendency to discount or ignore evidence and arguments that contradict one’s current beliefs. It really pervades all of our thinking, right down to our perceptions.

Because of confirmation bias, we need to train ourselves to recognize valid arguments for conclusions we think are false. Remember, an argument is valid if it is impossible for the premises to be true and the conclusion false. This means that you can have valid arguments with false conclusions, they just also have to have at least one false premise. Consider this example:

Oranges are either fruits or musical instruments.

Oranges are not fruits.

Oranges are musical instruments.

The conclusion of this argument is nonsensical. Nevertheless, it follows validly from the premises. This is a valid argument. If both premises were true, then the conclusion would necessarily be true.

**This shows that a valid argument does not need to have true premises or a true conclusion. Conversely, having true premises and a true conclusion is not enough to make an argument valid.** Consider this example:

London is in England.

Beijing is in China.

Paris is in France.

The premises and conclusion of this argument are, as a matter of fact, all true. This is a terrible argument, however, because the premises have nothing to do with the conclusion. The argument is not valid. If an argument is not valid, it is called **invalid**.

**In general the actual truth or falsity of the premises, if known, do not tell you whether or not an inference is valid**. There is one exception: when the premises are true and the conclusion is false, the inference *cannot* be valid, because valid reasoning can only yield a true conclusion when beginning from true premises.

Here is another invalid argument:

All dogs are mammals.

All dogs are animals.

All animals are mammals.

In this case, we can see that the argument is invalid by looking at the truth of the premises and conclusion. We know the premises are true. We know that the conclusion is false. This is the one circumstance that a valid argument is supposed to make impossible.

Some invalid arguments are hard to detect because they resemble valid arguments. Consider this one:

An economic stimulus package will allow the U.S. to avoid a depression.

There is no economic stimulus package.

The U.S. will not avoid a depression.

This reasoning is not valid since the premises do not definitively support the conclusion. To see this, assume that the premises are true and then ask, “Is it possible that the conclusion could be false in such a situation?” There is no inconsistency in taking the premises to be true without taking the conclusion to be true. The first premise says that the stimulus package will allow the U.S. to avoid a depression, but it does not say that a stimulus package is the only way to avoid a depression. Thus, the mere fact that there is no stimulus package does not necessarily mean that a depression will occur.

When an argument resembles a good argument but is actually a bad one, we say it has a **fallacy**. Fallacies are similar to cognitive biases, in that they are ways our reasoning can go wrong. Fallacies, however, are always mistakes you can explicitly lay out as arguments in canonical form, as above.

#### Soundness

If an argument is not only valid, but also has true premises, we call it **sound**. “Sound” is the highest compliment you can pay an argument. We said earlier that there were two ways an argument could go wrong, either by having false premises or weak inferences. Sound arguments have true premises and undeniable inferences. An argument that fails, either by having invalid logic, or by having at least one false premise, is called **unsound**.

This argument is valid, but not sound:

Socrates is a person.

All people are carrots.

Therefore, Socrates is a carrot.

This argument both valid and sound:

Socrates is a person.

All people are mortal.

Therefore, Socrates is mortal.

Both arguments have the exact same form. They say that a thing belongs to a general category and everything in that category has a certain property, so the thing has that property. Because the form is the same, it is the same valid inference each time. The difference in the arguments is not the validity of the inference, but the truth of the second premise. People are not carrots, therefore the first argument is not sound. People are mortal, so the second argument is sound.

Often it is easy to tell the difference between a valid but unsound argument, and a valid and sound argument, if you are using completely silly examples. Things become more complicated with false premises that you might be tempted to believe, as in this argument:

Every Irishman drinks Guiness.

Smith is an Irishman.

Therefore, Smith drinks Guiness.

You might have a general sense that this argument is bad—you shouldn’t assume that someone drinks Guinness just because they are Irish. But the argument is completely valid (at least when it is expressed this way). The inference here is the same as it was in the previous two arguments. The problem is the first premise. Not all Irishmen drink Guinness, but if they did, and Smith was an Irishman, he would drink Guinness.

The important thing to remember is that **validity is not about the actual truth or falsity of the statements in the argument**. Instead, it is about the way the premises and conclusion are put together. **It is really about the form of the argument**. A valid argument has perfect logical form. The premises and conclusion have been put together so that the truth of the premises is incompatible with the falsity of the conclusion.

The following table gives examples of both valid and invalid arguments with a variety of truth values for the premises and the conclusion. Notice that there are many ways for an argument to be unsound, but only one way to be sound. Notice also that the only time truth values can tell us anything about validity is when the premises are actually true and the conclusion is actually false. If an argument is valid, it is impossible to have true premises and a false conclusion, so if these are the real-world truth values, something went wrong with the logic, and the argument has to be invalid.

Table 1: Deductive Arguments

|  | Valid | Invalid |
| --- | --- | --- |
| True Premises  True Conclusion | All mammals are animals.  Cats are mammals.  Therefore, cats are animals.  (sound) | All mammals are animals.  Cats are animals.  Therefore, cats are mammals.  (unsound) |
| True Premises  False Conclusion | Does Not Exist | All mammals are animals.  Snakes are animals.  Therefore, snakes are mammals.  (unsound) |
| False Premises  True Conclusion | All mammals are fruits.  Bananas are mammals.  Therefore, bananas are fruits.  (unsound) | All mammals are fruits.  Cats are fruits.  Therefore, cats are mammals.  (unsound) |
| False Premises  False Conclusion | All mammals are shoes.  Bananas are mammals.  Therefore, bananas are shoes.  (unsound) | All mammals are fruits.  Snakes are fruits.  Therefore, snakes are mammals.  (unsound) |

A general trick for determining whether an argument is valid is to try to come up with just one way in which the premises could be true but the conclusion false. If you can think of one, the reasoning is invalid. This is called the counterexample method (more on that below).

### Inductive Arguments

Instead of “valid” and “invalid,” when we evaluate the logic of inductive arguments, we use the terms “strong” for good arguments and “weak” for bad ones.

#### Strength

Inductive arguments, remember, can never guarantee their conclusion; there’s always room for error. So we can’t really apply “validity” or “invalidity” to them.[[5]](#footnote-5) Inductive arguments can, however, give evidence that the conclusion is probably. We evaluate them according to how probable the premises make the conclusion—how strongly the premises support the conclusion. For the purposes of this textbook we will adopt the following definitions of strength:

Strong: Conclusion is made probable in light of the premises. So: *if* the premises are true, the conclusion is probably true. The premises give good support for the conclusion.

Weak: Conclusion is NOT made probable in light of the premises; i.e., the premises give little or no support for the conclusion.

An argument is **strong** if the premises would make the conclusion more probable, were they true. In a strong argument, the premises don’t guarantee the conclusion, but they do make it a good bet—we say that they make the conclusion *probable.*

You may have noticed that the word “probable” is a little vague. How probable do the premises have to make the conclusion before we can count the argument as strong? The answer is a very unsatisfying “it depends.” As a general rule, any argument where the premises give the conclusion a probability greater than 50% is a strong argument. We don’t often have actual numbers support our inductive arguments, so we have to use common sense. Take the following argument:

The highway sign says “Atlanta, 34 miles.” Therefore, it’s probably 34 miles to Atlanta.

This is an argument based on signs. We don’t possess statistics on how frequently highway signs are accurate, but common sense tells us that if they weren’t accurate more than half the time there’d be no point to having them at all.

The vagueness of the word “probable” brings out an interesting feature of strong arguments: some strong arguments are stronger than others. Consider the following two arguments:

My friend Tessa has the iPhone Infinity and they love it. I bet if I get the iPhone Infinity, I will also love it.

My friends Tessa, Lester, George, and Imogen have the iPhone Infinity and they love it. I bet if I get the iPhone Infinity, I will also love it.

The argument gives only one instance of another iPhone user loving their phone. The argument is not very strong as it stands but it can become stronger if we add more users as examples. The more evidence we have, the better a bet the conclusion is.

When we do induction, we try for strong inferences, where the premises, assuming they are true, would make the truth of the conclusion highly probable, though not necessary. Consider these two arguments. First, look at this one:

92% of Republicans from Texas voted for Bush in 2000. Jack is a Republican from Texas, so Jack probably voted for Bush.

This is a strong argument. Now compare it to this one, which is weak:

One half of all drivers are men. There’s a person driving the car that just cut me off, so the person driving that car is a probably a man.

There is a big difference between how much support the premises, if true, would give to the conclusion in the first and how much they would in the second. The premises in the first, assuming they are true, would provide very strong reasons to accept the conclusion. This, however, is not the case with the second: if the premises in it were true then they would give only weak reasons for believing the conclusion. Thus, the first is strong while the second is weak.

#### Cogency

A cogent argument is the inductive equivalent of sound deductive argument: it’s a strong inductive argument with true premises. Cogent equals strong, plus all true premises. Always think of the definition in that order. First determine whether the argument is strong, and then ask whether the premises are true. It’s also important that the “true premises” condition requires that you not leave out any important evidence that would tend to weigh against the conclusion. For example, if it’s a warm, sunny day, with only gentle breezes and no big waves, you may conclude it’s a nice day for swimming in the ocean. That would typically be a strong argument. But if you’ve ignored the fact that a large ship just dumped huge quantities of toxic waste near the beach, your argument is weak. This is often referred to as the *total evidence requirement*. In sum, then, an argument is cogent if it is strong and has true premises. It is uncogent if it is weak, or has false premises.

The following table gives examples of strong and weak inductive arguments, with a variety of truth values for the premises and conclusions.

Table 2 Inductive Arguments

|  | Strong | Weak |
| --- | --- | --- |
| True Premise  Probably True Conclusion | All previous Halloween’s were celebrated in October.  Probably the next Halloween will be celebrated in October.  (cogent) | Several Halloweens were celebrated in the 19th century.  Probably the next Halloween will be celebrated in October.  (uncogent) |
| True Premise  Probably False Conclusion | Does Not Exist | Several Halloweens were celebrated in the 19th century.  Probably the next Halloween will be celebrated in January.  (uncogent) |
| False Premise  Probably True Conclusion | All previous Halloweens were celebrated in the 21st century.  Probably the next Halloween will be celebrated in the 21st century  (uncogent) | Several Halloweens were celebrated in January.  Probably the next Halloween will be celebrated in October.  (uncogent) |
| False Premise  Probably False Conclusion | All previous Halloweens were celebrated in January.  Probably the next Halloween will be celebrated in January.  (uncogent) | Several Halloweens were celebrated in January.  Probably the next Halloween will be celebrated in January.  (uncogent) |

## IV Counterexample Method

It has been stated repeatedly that validity is entirely a function of the form of a deductive argument, and that the actual truth values of the statements involved are irrelevant. The following argument, for example, is valid:

All cats are dogs.

All dogs are fish.

Therefore, all cats are fish.

Since validity is determined by the form of the argument, it follows that any argument with the same form will also be valid. The form of the above argument is:

All C are D

All D are F

All C are F

Similarly, the following argument is invalid because it has an invalid form:

Some cats are mammals

Some cats are animals.

Some mammals are animals.

Everything in this argument is true, so it would be tempting to think this is a valid argument. It’s not, though, because it’s possible for an argument with true premises and a false conclusion to have this same form. Any argument where the form would allow true premises and a false conclusion is an invalid argument. Here’s the form of the argument immediately above:

Some C are M

Some C are A

Some M are A

The following argument has the very same form, but has true premises and a false conclusion. Table 1 above tells us that no valid argument form would allow that.

Some animals are cats

Some animals are dogs

Some cats are dogs

Again, the question is not whether the premises and conclusion are true or false, but whether the premises *support* the conclusion. That’s what table 1 above illustrates. The validity of a deductive argument, to put it another way, is determined solely by the original argument’s form. If that form would allow true premises and a false conclusion, then the form is invalid. Thus, there is only one empty box in table 1—the box representing a valid argument with true premises and a false conclusion. Every other arrangement of truth values is possible for both valid and invalid arguments—true premises and true conclusion, false premises and true conclusion, false premises and false conclusion. But no valid argument form will ever allow true premises and a false conclusion.

This seems counter-intuitive, but you can have a *valid* deductive argument in which everything is false, so long as the conclusion actually follows from the premises. And you can have an *invalid* deductive argument in which everything is true, so long as the conclusion does not follow from the premises. And whether the conclusion does follow from the premise will be determined by the argument’s ***form***—the arrangement of the terms in the argument, and not what the argument is about.

In order to *prove* that a deductive argument is invalid, we have been using the counterexample method. In the counterexample method we look for a substitution instance for the terms in the original argument so that the premises come out true and the conclusion false. Here are the steps:

1. Write out the original argument.
2. Isolate the form of the original argument.
3. Find terms to substitute for those in the original argument.
4. Plug substitute terms into the form so that you make the premises true and the conclusion false.

**1. Original argument**

Some humans are doctors.

Some humans are men.

Therefore, some men are doctors.

This is a categorical syllogism. All the statements in it are true, so it would be tempting to think that it is valid. This would be a mistake. This argument is invalid, but it’s hard to see that until you break it down into its form and then do substitutions for the terms. Here’s the form of the original argument:

**2. Form of the original argument**:

Some H are D

Some H are M

Some M are D

**3. Find substitute terms for those in the original argument**

H = Animals

D = Cats

M = Dogs

**4. Plug substitute terms into the original argument form**

Some Animals are Cats (True)

Some Animals are Dogs (True)

Some Dogs are Cats (False)

*Note about “terms” used for substitutions*. For categorical syllogisms, the terms you use for substitutions must be plural nouns (dogs, cats, animals, mammals) or plural noun phrases (dogs who like to watch football, cats who smoke cigarettes, mammals riding bicycles).

### Counterexample Method and non-Categorical Syllogisms

The substitution method can also be used for hypothetical syllogisms, disjunctive syllogisms and more—and follows the same steps. You need to find substitutions that make the premises true and the conclusion false. For hypothetical and disjunctive syllogisms, what you need to use for substitutions are full statements, and not just terms like we used with categorical syllogisms. Remember, the steps are:

1. Write out the original argument.
2. Isolate the form of the original argument.
3. Find statements to substitute for those in the original argument.
4. Plug substitute statements into the form so that you make the premises true and the conclusion false.

**1. Original argument***:*

If Beyoncé is a cat, then Beyoncé is a feline. (True)

Beyoncé is not a cat. (True)

Therefore, Beyoncé is not a feline. (True)

**2. Original argument form**:

If C then F

Not C

Not F

This is an invalid argument, even though everything in it is true. We can prove it’s invalid by substituting the following *statements* for C and F (you can always use these two statements to check invalidity for hypothetical syllogisms).

**3. Find substitute terms for those in the original argument**

C = Atlanta is the capital of Florida

F = Atlanta is a southern city

**4. Plug substitute terms into the original argument form**

If (C) Atlanta is the capital of Florida, then (F) Atlanta is a southern city. (True)

(not C) Atlanta is not the capital of Florida. (True)

(not F) Atlanta is not a southern city. (False)

### Summing Up

A valid deductive argument form will not allow you to substitute terms or statements to make the premises true and have the conclusion come out false. If you *can* do that, find substitute terms or statements for the terms in the original argument that make the premises true and the conclusion false, then you have proved that the original argument has an invalid form, i.e., you have proved that the original argument is invalid.

1. This chapter is based on *For All X, The Lorain County Remix*, remixed by J. Robert Loftis. [↑](#footnote-ref-1)
2. Peter Singer, “Famine, Affluence, and Morality.” *Philosophy and Public Affairs*, Spring, 1972, Vol. 1 No. 3. [↑](#footnote-ref-2)
3. If you are reading this with a screen reader, there is a line between the last and second-to-last statements. In canonical form, we will always put the conclusion last, so you should be able to find the conclusion without knowing the line is there. [↑](#footnote-ref-3)
4. Aristotle *On the Heavens*, 298a2-10. [↑](#footnote-ref-4)
5. I mean, you can. But given inductive arguments always leave room for error, according to the definitions of validity and invalidity, all inductive arguments are invalid. This is hardly helpful in evaluating them, however—some inductive arguments are much better than others, and we need to be able to tell the difference. [↑](#footnote-ref-5)