One Dimensional Motion

The Inclined Plane and Kinematic Equations

Background

According to Newton's first law, an object moving without the influence of a net external force maintains a constant velocity. However, an object that experiences a net external force does accelerate.

In Figure 1, an object is moving along a *frictionless* inclined plane, with angle of inclination θ. The only force acting on the object is force due to gravity, which by Newton's Second Law of Motion is given by

$$\left|\left|\vec{F}\right|\right| = mass \cdot |g| \text{ Newtons.}$$

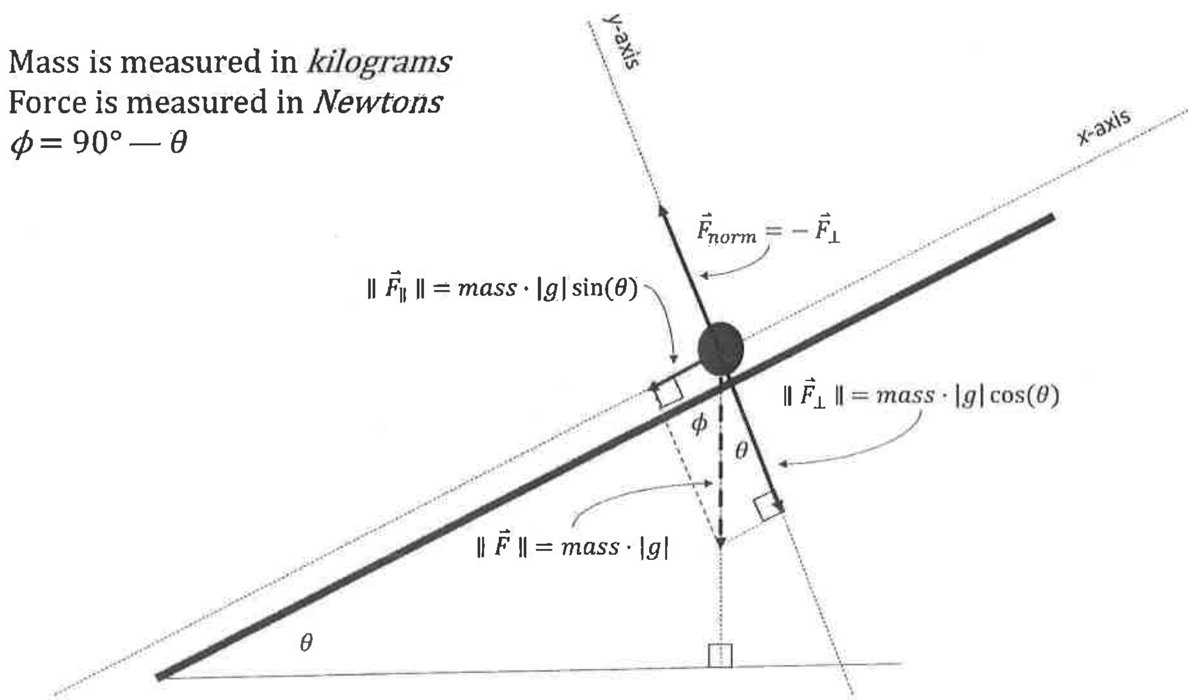

Figure 1: Inclined Plane Schematic for Deriving Formulas

Setting our reference frame so that the origin goes through the center of the object, with the *x*-axis parallel to the inclined plane, we can decompose force due to gravity into *x* and *y* components:

$$\left\|\vec{F}_{||}\right\| = mass \cdot |g| \cdot \sin(\theta) \text{ Newtons;}$$

$$\left\|\vec{F}_{\perp}\right\| = mass \cdot |g| \cdot \cos(\theta) \text{ Newtons.}$$

By Newton's Third Law of Motion, there is an equal and opposite force $\vec{F}_{norm} = -\vec{F}_{\perp}$ acting on the object since there is no vertical motion.Since there is no force in the positive *x* direction, the net force acting on the object is

$$\vec{F}_{net} = \vec{F}_{||} + \vec{F}_{\perp} + \vec{F}_{norm} = \vec{F}_{||} \text{ Newtons.}$$

Revisiting Newton's Second Law of Motion, because the object accelerates down the inclined plane, we see

$$\left\|\vec{F}_{net}\right\| = \left\|\vec{F}_{||}\right\| = mass \cdot (|g| \cdot \sin(\theta)).$$

tells us that the *acceleration along the inclined plane* is $a_x = |g| \cdot \sin(\theta)$ meters per second squared; note that this acceleration is *positive* and *constant*.

Kinematic Equations

The position $x(t)$ of the object is given by the *kinematic equation*:

$$x(t) = \frac{1}{2}a_x t^2 + v_{0x}t + x_0 \text{ meters (Kinematic Equation 1).}$$

Taking the initial position of the object to be the origin and substituting what we know to be the acceleration in the *x*-direction, and assuming that the initial velocity in the *x*-direction is 0 meters per second, we have

$$x(t) = \frac{1}{2}(|g| \cdot \sin(\theta)) \cdot t^2 \text{ meters (Equation 1A).}$$

Because the object accelerates (net force is non-zero), the velocity at any time *t* is given by

$$v(t) = a_x t + v_{0x} \text{ meters per second (Kinematic Equation 2).}$$

In our case, this becomes

$$v(t) = (|g| \cdot \sin(\theta)) \cdot t \text{ meters per second (Equation 2A).}$$

Depending upon what values are known and which are unknown, two additional kinematic equations can be derived from the two given so far.

Using Kinematic Equation 1, we can write

$$x(t) - x_0 = \left(\frac{1}{2}a_x t + v_{0x}\right) t$$

$$\Delta x = \left(\frac{1}{2}\left(v(t) - v_{0x}\right) + v_{0x}\right) t$$

$$\Delta x = \left(\frac{v(t) - v_{0x} + 2v_{0x}}{2}\right) t$$

$$\Delta x = \left(\frac{v(t) + v_{0x}}{2}\right) t \text{ (Kinematic Equation 3)}$$

Finally, we can create an equation that relates velocity, acceleration, and position in a slightly different way:

$$\Delta x = \left(\frac{v(t) + v_{0x}}{2}\right) t$$

$$\Delta x = \left(\frac{v(t) + v_{0x}}{2}\right)\left(\frac{v(t) - v_{0x}}{a_x}\right)$$

$$\Delta x = \left(\frac{v(t) + v_{0x}}{2}\right)\left(\frac{v(t) - v_{0x}}{a_x}\right)$$

$$\Delta x = \frac{v^2(t) - v^2_{0x}}{2a_x}$$

$$2\, a_x \Delta x = v^2(t) - v^2_{0x}$$

$$v^2(t) = v^2_{0x} + 2\, a_x \Delta x \text{ (Kinematic Equation 4)}$$

Lab 4 Objectives

A cart is released from rest and allowed to travel undisturbed down an inclined frictionless track. Using collected data, students will experimentally verify Equations 1A and 2A using PASCO Capstone software by generating two graphs—a position versus time graph and a velocity versus time graph. Error analysis will be performed on the coefficient of t^2 in Equation 1A and the coefficient of t in Equation 2A by observing the percent error between the two values and the expected coefficients from kinematic equations 1 and 2. Finally, the two experimental coefficients will be compared using percent difference and the third and fourth kinematic equations will be checked against experimental values.

Additionally, students will become more familiar with using Capstone to create graphs, apply models to graphs, and to copy the graphs to a Google Doc.

Materials and Equipment

For each student or group:

- GLX data collection system
- Laptop running PASCO Capstone software
- USB cable to connect GLX to the laptop
- End stop
- Large base and support rod
- Dynamics track rod clamp
- Dynamics cart
- Ruler
- Protractor
- Thumb drive (optional)

Safety

Follow all standard laboratory safety features. Note that the dynamics cart should be positioned correctly on the track to avoid the cart falling off the track.

The angle of inclination should not exceed 30° or the cart may damage the end stop or loosen it.

Procedure

Note*: The instructions given here correspond to the PASCO data collection system. There is a copy of the instructions for the entire device in the lab. It is far more sophisticated than we will encounter in this laboratory.*

Part I: Set Up

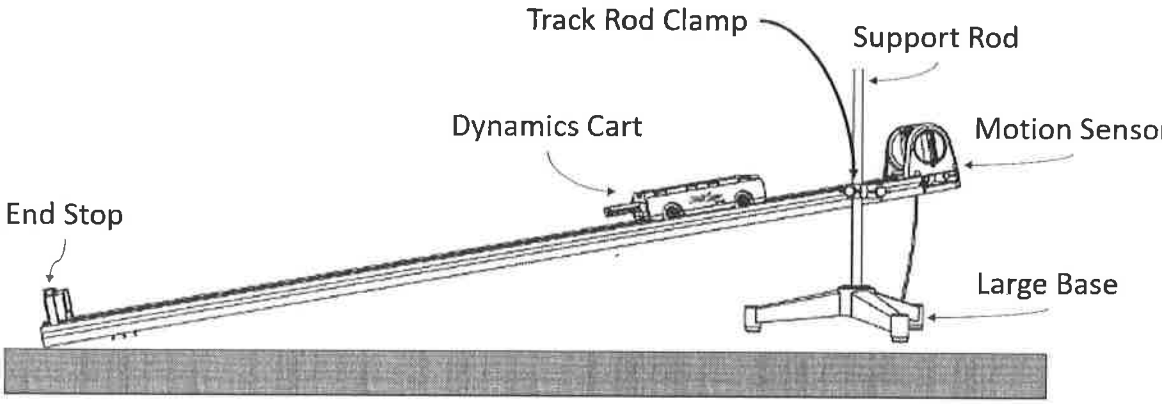

Figure 2: Incline Plane Set Up

I.1. Attach one end of the dynamics track to the large base and support rod using the dynamics track rod clamp, inclining the track slightly (Figure 2).

I.2. Mount the motion sensor to the inclined end of the track with the sensing element on the sensor pointing down the length of the track. Make certain the switch on the top of the sensor is set to the cart icon.

I.3. Mount the end stop to the bottom end of the track.

I.4. Connect the motion sensor to the data collection system.

I.5. Connect the motion sensor to the first port on the GLX:

Figure 2: The GLX Data Collection Device Ports

I.6. Connect the GLX to the laptop using the USB cable.

I.7. Open the file PHYS210_Lab4.cap. Make sure that the motion sensor is the selected sensor on the worksheet and that sampling is set to 20 Hz (Hertz = cycles per second). See Figure 3.

Figure 3: Capstone Sensor Recording Set Up

Part II: The Experiment

II.1. Use the protractor to estimate, to the nearest degree, the angle of inclination θ. Record the value below:

$\theta =$ ____________ °

II.2. Compute $a_x = 9.81 \cdot \sin(\theta)$ and write the value below (2 significant figures):

$a_x \approx$ ____________________ m/s^2

II.3. Compute $\frac{1}{2} \cdot 9.81 \cdot \sin(\theta)$ and write the value below (2 significant figures):

$\frac{1}{2}a_x \approx$ ____________________ m/s^2

II.4. Tap the calculator icon on the left side of the screen in Capstone and enter the measured estimate for the angle of inclination as **theta**. Compare your result in II.2 with the computed result in the Capstone calculator. Tap the calculator icon again to close the calculator.

II.5. Tap the data display icon on the left side of the screen and then choose the cog and then click the “Zero Sensor Now” button to tare the motion sensor (Figure 4).

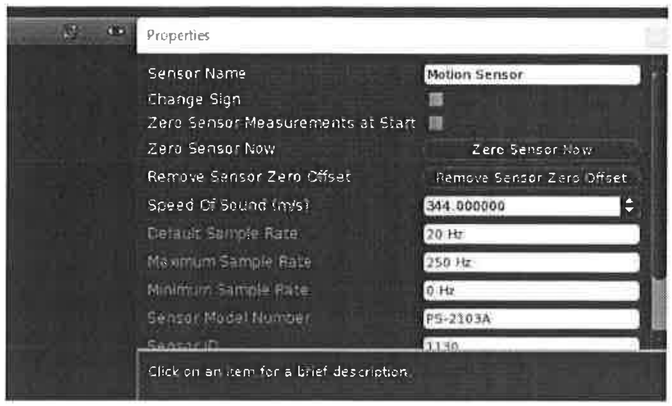

Figure 4: Tare the Motion Sensor

II.6. Hold the dynamics cart stationary on the track approximately 10 cm in front of the motion sensor. Make sure that the end of the cart with the plunger is pointed away from the motion sensor and that the plunger is pushed in, flush with the front of the cart.

II.7. Press the record button, wait for data recording to begin, and release the cart.

II.8. As soon as the cart has reached the end of the track and *before* it hits the end stop, press the record button again to stop collecting data. Don't worry if you miss it; it's better to stop too late than to stop too early.

II.9. Have your instructor view your data table to see if you need to run the experiment again.

II.10. Delete any rows of data at the bottom with *negative* velocity (these indicate the cart is rolling up the plane, not down) and the rows beneath these. If, at the top, there are repeated instances of position and velocity both being 0 units, delete all but one of these rows, as well (there should be some, since you started recording before releasing the cart). Keep track of how many rows you delete, as this should be part of your discussion:

Number of deleted rows of data = ____________ rows

II.11. Locate the last row in the data table and record the *elapsed time*, *final position* and *final velocity* in the table, and copy their values below:

elapsed time $t_f =$ ____________________ seconds

final position $x(t_f)$ = ____________________ meters

final velocity $v(t_f)$ = ____________________ meters per second

II.12. Using the indicated kinematic equations, estimate the acceleration by solving for a_x:

a. $x(t) = \frac{1}{2}a_x t^2 + v_{0x}t + x_0 \Rightarrow x_f = \frac{1}{2}a_x t_f^2 \Rightarrow a_x = \frac{2x_f}{t_f^2} \approx$ ___ m/s²

b. $v(t) = a_x t + v_{0x} \Rightarrow v_f = a_x t_f \Rightarrow a_x = \frac{v_f}{t_f} \approx$ ___ m/s²

c. $v^2(t) = v^2_{0x} + 2\,a_x \Delta x \Rightarrow v_f^2 = 2a_x x_f \Rightarrow a_x = \frac{v_f^2}{2a_x} \approx$ ___ m/s²

II.13. On each graphics display, click the button in the upper left corner of the display to scale the axes to include all data points.

II.14. On the Position vs Time graph, select the curve fitting tool and choose a quadratic fit; copy the fitted equation, with appropriate significant figures, below:

$x(t) =$ ____ meters

II.15. Record the coefficient of t^2 below:

$\frac{1}{2}a_x \approx$ ________________________________ m/s²

II.16. Find the *percent error* between the value in Step 10 with the theoretical value in Step II.3.

$\frac{|experimental - theoretical|}{theoretical} \times 100\% =$ ______________%

II.17. On the Velocity vs Time graph, select the curve fitting tool and choose a linear fit. Copy the fitted equation, with appropriate significant figures, below:

$v(t)$ = ______________________________ meters per second

II.18. Record the coefficient of t below:

$a_x \approx$ ______________________________ m/s^2

II.19. Find the *percent error* between the value in Step 10 with the theoretical value in Step II.2.

$$\frac{|experimental - theoretical|}{theoretical} \times 100\% = \_\_\_\_\_\_\_\_\_\_\_\_\_ \%$$

II.20. Save your experiment on the laptop (or a flash drive).

II.21. Open Google Chrome and log into https://drive.google.com with your Salem credentials.

II.22. Type *docs.new* to open a new Google Doc.

II.23. Return to Capstone and click on a blank space in the Position vs Time graph. In the Display menu, choose "Copy Display" and paste it into your Google Doc.

II.24. Repeat Step II.19 for the Velocity vs Time graph.

II.25. Share your Google Doc with each of your lab partners.

II.26. Log out of Google Chrome and close the browser.

II.27. Open the calculator  in Capstone. Double click on each of the constants θ, t_f, x_f, v_f and enter your experimental values (Steps II.1 and II.11) after the = sign. Tap the calculator icon again to close it. You can use the computed values in the calculator later, in the Analysis section, to check your computations.

Analysis

Note: Both graphs need to be included in your laboratory report. The table below should also be included in your analysis section.

Analysis Questions

Make sure that your analysis incorporates the answers to all of the following questions in a narrative form.

1. Using your estimate for acceleration (choose the one with the least amount of percent error), along with $v_{0x} = 0.0$ m/s, $x_0 = 0.0$ m, θ, and t_f, $x(t_f)$, $v(t_f)$ from Step II.11 in the Procedure section, complete the table below for the third and fourth *kinematic equations* and check your answers with those computed by the Capstone calculator:

Table 1: Checking the Kinematic Equations

Equation	Left-Hand Side	Right-Hand Side	Percent Difference
$\Delta x = \left(\frac{v(t)+v_{0x}}{2}\right) t$	$x(t_f) =$ ________ m	$\frac{v(t_f)}{2} \cdot t_f =$ ________ m	
$v^2(t) = v^2_{0x} + 2\, a_x \Delta x$	$v^2(t_f) =$ ______ $(m/s^2)^2$	$2\,\lvert g\rvert \sin(\theta)\, x(t_f)$ ______ $(m/s^2)^2$	

2. Consider the approximations for a_x you computed in II.12. Find the *percent difference* for each of these approximations and the

3. Why do you think that the position vs. time graph is modeled by a concave *up* parabola rather than a concave *down* parabola? (Hints: Concavity is determined by the coefficient of t^2 here; the *x*-axis is positive in the direction of the motion of the cart; the velocity is increasing with time; we are looking at horizontal change in position, rather than vertical.)

4. What does the slope of the linear best fit line represent in your graph of velocity versus time for the cart traveling down an incline?

5. What does the vertical intercept of the linear fit line represent in your graph of velocity versus time for the cart traveling down an incline?

Discussion

Write at least two paragraphs for your conclusion, which needs to include, but is not limited to:

1. What physical relationships between position and time, and velocity and time of an object subjected to constant acceleration down an inclined plane have you learned from this experiment? Do you think the angle of the incline had an effect on your estimated acceleration? If so, why?

2. Did your experimental values provide reasonable validation of the kinematic equations? Why or why not?

3. What kind of errors were encountered during the experiment? Use the "Types of errors" from Lab 1 to help you to identify the errors.