**Lesson 1 Notes: Problem Solving Strategies**

**ANSWER KEY**

*The primary goal of this class is to challenge you to think like a Mathematician!*

The Common Core State Standards for Mathematics identifies eight “Mathematical Practices.” The first practice is:

* Make sense of problems and persevere in solving them
* Our goal is to help you develop skills necessary to solve problems

Part of solving a problem is:

* Understanding what is being asked
* Knowing what a solution should look like

**Problem or Exercise?**

* In a PROBLEM, you probably do not know how to approach solving it.
* In an EXERCISE, you are often practicing a skill.

What is a problem for some students may be an exercise for another!

***Problems often involve false starts, making mistakes, and lots of scratch paper!***

**For each question below, decide if it is a**problem**or an**exercise**. (You do not need to solve the problems! Decide which category fits for you.)**

PROBLEM

**Problem Solving Strategies**

* There is never a simple recipe for problem solving.
* You get better by building background knowledge, practicing, learning from others and getting it wrong.
* No single strategy works every time.

1) Wishful Thinking

2) Try Something

3) Draw a Picture

4) Make Up Numbers

5) Try A Simpler Problem

6) Work Systematically

7) Use Manipulatives to Investigate

8) Look for and Explain Patterns

9) Find Math and Remove Content

10) Check your Assumptions

EXERCISE

1. What is the sum of 125 and 30?

Problem or Exercise: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2. This clock has been broken into three pieces. If you add the numbers in each piece, the sums are consecutive numbers. (Note: Consecutive numbers are whole numbers that appear one after the other, such as 1, 2, 3, 4 or 13, 14, 15.)

Problem or Exercise: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

### Problem or Exercise: \_\_ Problem\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Polya’s Four-Step Method for Problem Solving**

1. Understand the Problem
2. Make a Plan
3. Carry out the Plan
4. Look back over your work. How could it be better?

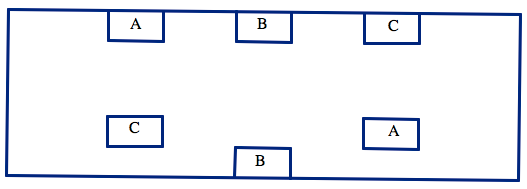
George Polya taught effective problem-solving skills.

“How to Solve It” was George Polya’s most famous book.

**Problem Solving Strategy 1** (Wishful Thinking)

**Problem 1 (ABC)**

Draw curves connecting A to A, B to B, and C to C.  Your curves cannot cross or even touch each other, they cannot cross through any of the lettered boxes, and they cannot go outside the large box or even touch its sides.



Do not be afraid to change the problem! Ask yourself “what if” questions:

* What if the picture was different?
* What if the numbers were simpler?
* What if I just made up some numbers?

[**Watch Video!**](https://www.youtube.com/watch?v=aoZ0qSruJC0&t=9s)

**The Most Important Problem-Solving Strategy**

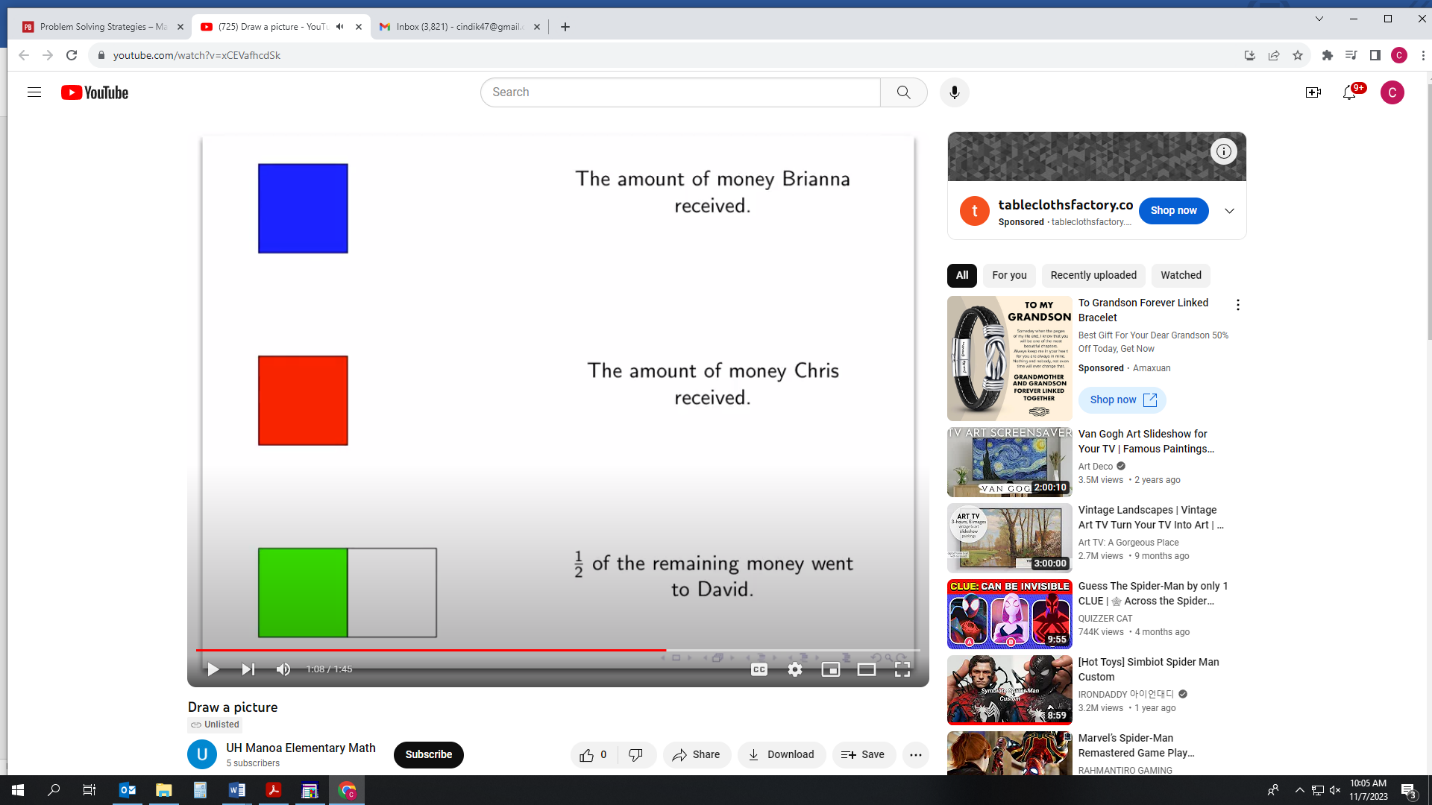
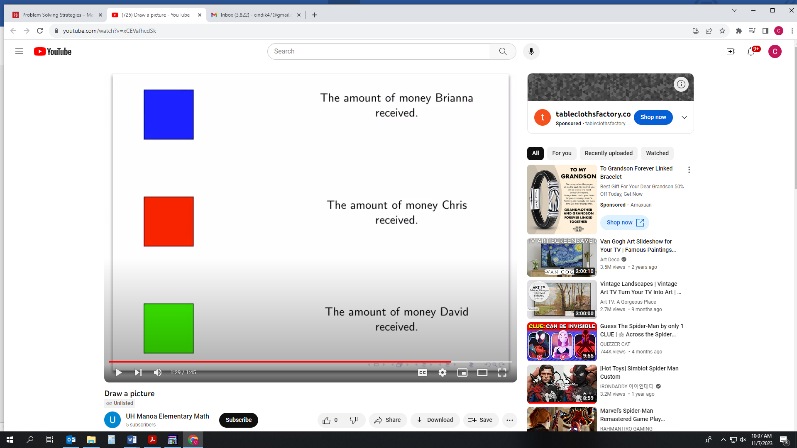
* TRY SOMETHING!
* The whole point of problem solving is that you do not know what to do right out of the starting gate.

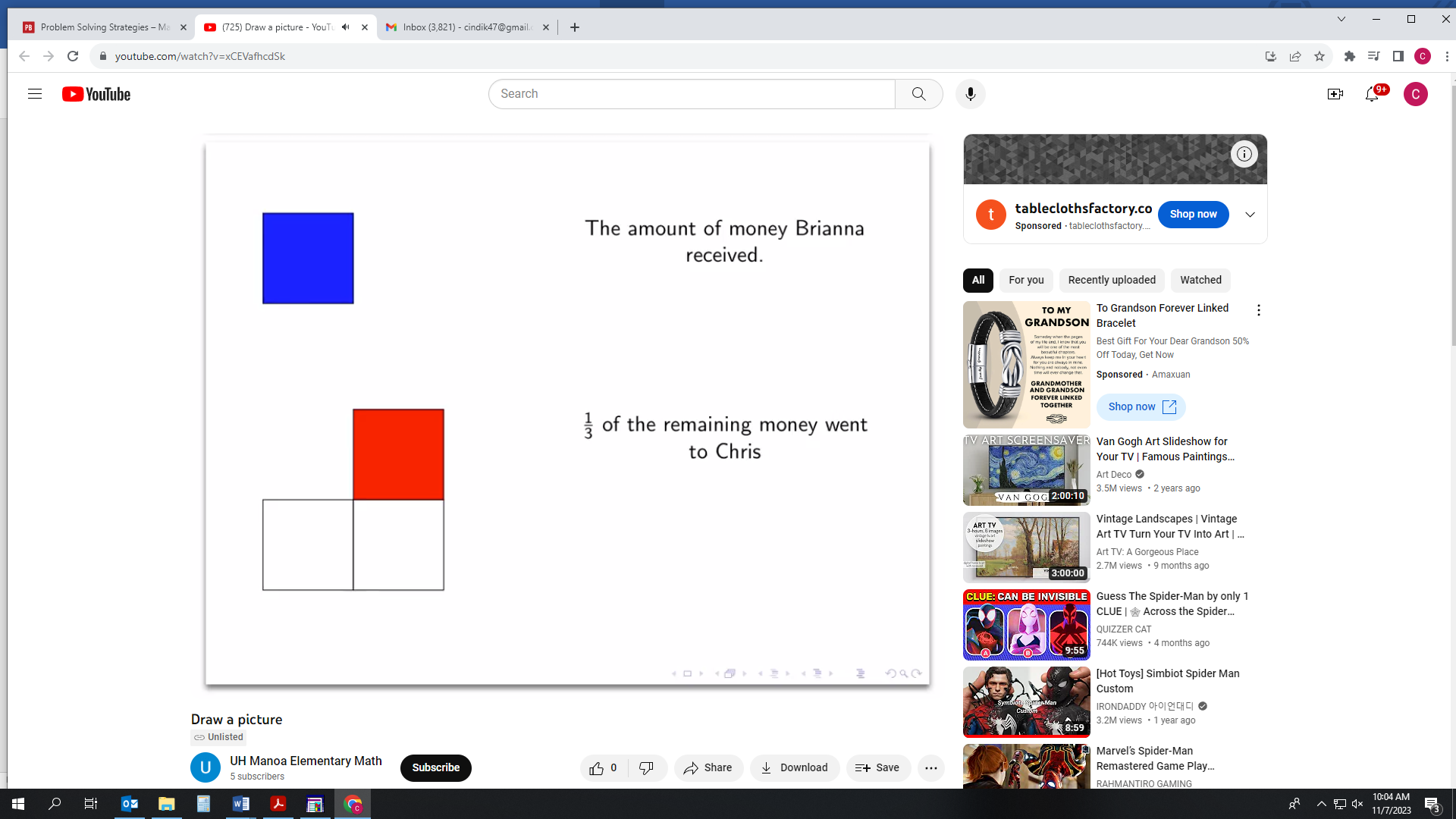
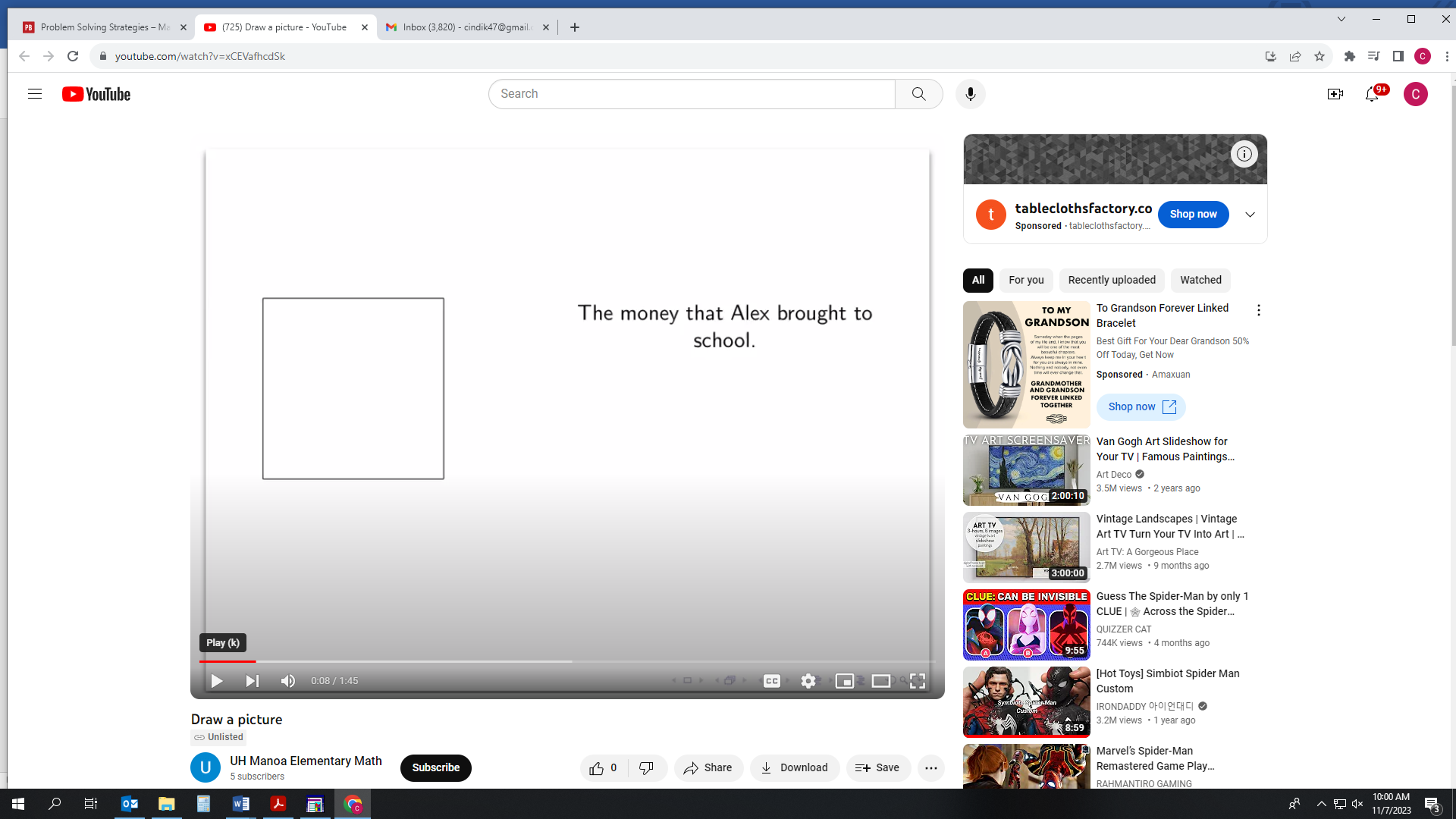
IF IT DOESN’T WORK, TRY SOMETHING ELSE.

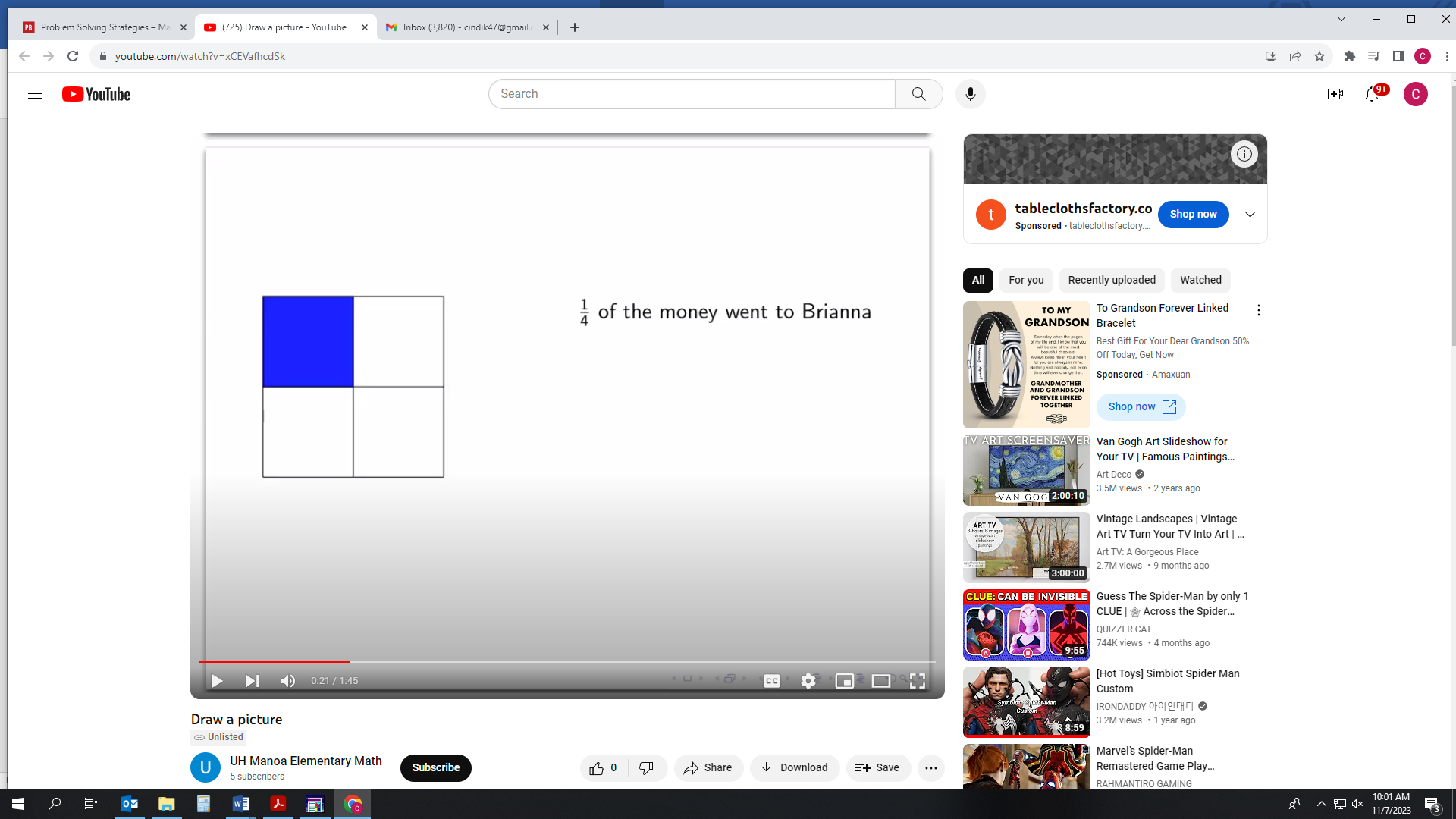
**Problem Solving Strategy 3** (Draw a Picture).

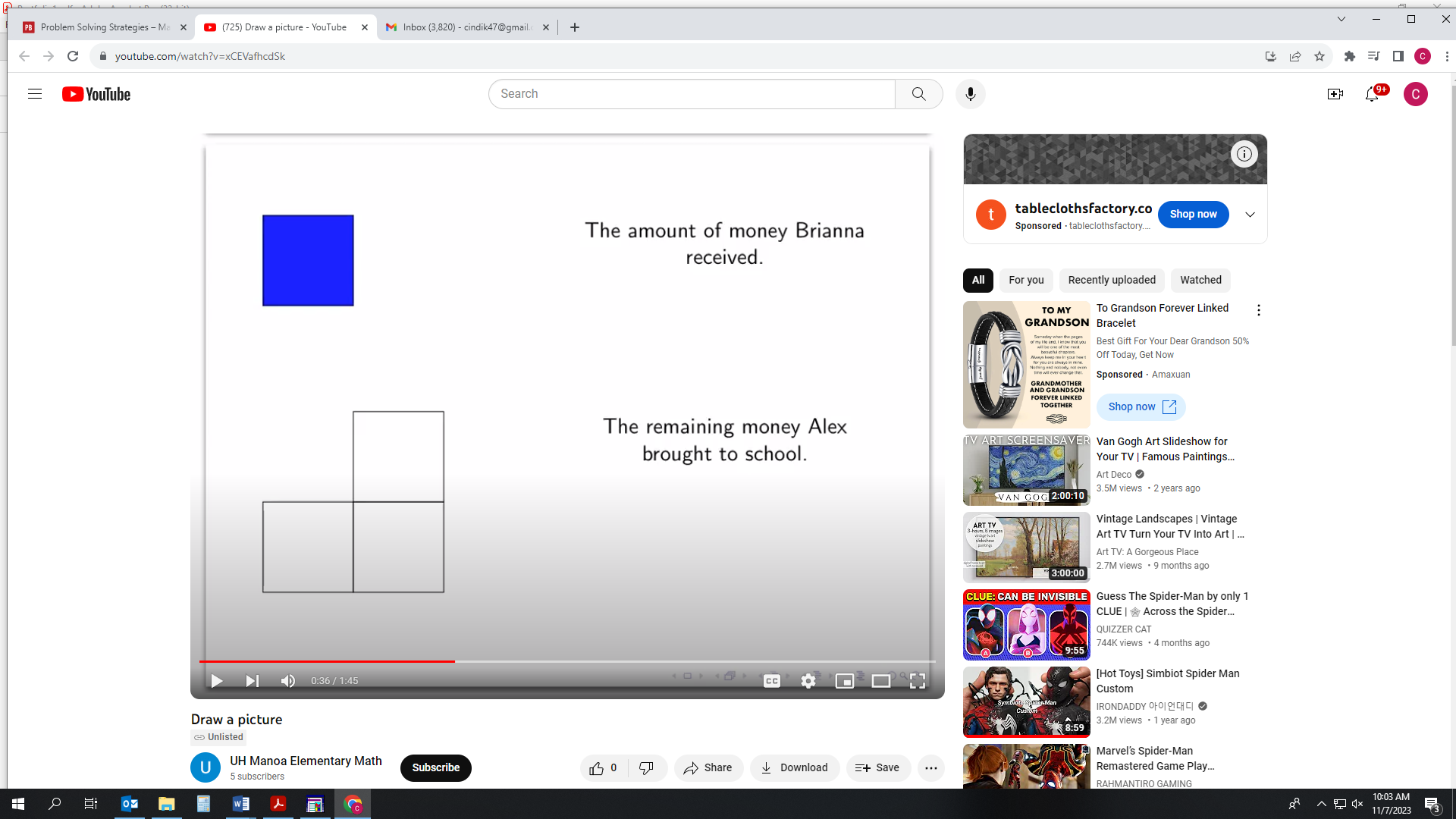
### Problem 2 (Payback)

Last week, Alex borrowed money from several of his friends. He finally got paid at work, so he brought cash to school to pay back his debts. First, he saw Brianna, and he gave her 1/4 of the money he had brought to school. Then Alex saw Chris and gave him 1/3 of what he had left after paying Brianna. Finally, Alex saw David and gave him 1/2 of what he had remaining. Who got the most money from Alex?

**Can you represent something in the situation by a picture?** [**Watch Video**](https://youtu.be/xCEVafhcdSk)







**Problem Solving Strategy 4** (Make Up Numbers).  You can work forwards: Assume Alex had some specific amount of money when he showed up at school, say $100. Then figure out how much he gives to each person. Or you can work backwards: suppose he has some specific amount left at the end, like $10. Since he gave David half of what he had left, that means he had $20 before running into David. Now, work backwards and figure out how much each person got. What’s the answer?

**Start with a fixed amount**

(I choose $100)

Alex sees Brianna and gives her ¼ of $100 = $25

Now Alex has $100 – 25 = $75

Alex sees Chris and gives him 1/3 of $75 = $25

Now Alex has $75 - 25 = $50

Alex gives David ½ of $50 which = $25

They all got the same amount!

**Work Backwards**

Assume Alex has $10 left.

He gave ½ of his money to David. He had to have had $20 when he met David.

(So, he had $20 plus the $10 he had left at the end for a total of $30 before meeting Chris.)

Alex has $30 and gives 1/3 of that to Chris which was $10.

So, before he met Chris he had $30 plus $10 for a total of $40. He gave Brianna ¼ which would be $10.

They all got the same amount!

### Problem 3 (Squares on a Chess Board)

How many squares, of any possible size, are on a 8 × 8 chess board? (The answer is not 64… It’s a lot bigger!)

**Explain Problem Solving Strategy 5** (Try a Simpler Problem) using the above problem.

* If you cannot solve the proposed problem, try to solve first some related problem.
* An 8 × 8 chess board is rather big. Can you solve the problem for smaller boards? Like 1×1? 2×2? 3×3?

**Problem Solving Strategy 6** (Work Systematically).

For example, in this problem you might keep track of how many 1 × 1 squares are on each board, how many 2 × 2 squares on are each board, how many 3 × 3 squares are on each board, and so on. You could keep track of the information in a table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **size of board** | **# of 1 × 1 squares** | **# of 2 × 2 squares** | **# of 3 × 3 squares** | **# of 4 × 4 squares** | **…** |
| **1 by 1** | 1 | 0 | 0 | 0 |  |
| **2 by 2** | 4 | 1 | 0 | 0 |  |
| **3 by 3** | 9 | 4 | 1 | 0 |  |

**Try it for an 8 X 8 chessboard?**

01 + 12 + 22 + 32 + 42 + 52 + 62 + 72 + 82 = 0 + 1 + 4 + 9 +16 + 25 + 36 + 49 + 64 = 204

**What if we were trying to figure out the number of possible squares for a 3 X 3 chessboard?**

0 x 0 = 0 squares (0)

1 x 1 = one 1 x 1 square + zero 0 x 0 squares (1 + 0)

2 x 2 = four 1 x 1 squares + one 2 x 2 square + zero 0 x 0 squares (4 + 1 + 0)

3 x 3 = nine 1 x 1 squares + four 2 x 2 squares + one 3 x 3 square + zero 0 x 0 squares (9 + 4 + 1 + 0)

If n = 3, the number of squares on your n x n grid is: 02 + 12 + 22 + 32 or 0 + 1 + 4 + 9 = 14.

You sum the squares from 0 to n.

***“It’s not that I’m so smart; it’s just that I stay with problems longer.” (Albert Einstein)***

Even Albert Einstein was wrong! But, the secret to being a good mathematician is to keep trying.

**Manipulatives**

Manipulatives are any concrete objects that allow students to explore an idea in an active, hands-on approach.

* Examples- Base ten blocks, counters, number lines, geoboards, coins, pattern blocks, fraction circle and much, much more.

**Problem Solving Strategy 7** (Use Manipulatives to Help You Investigate).

* Drawing a picture may not be enough!
* Having actual materials to move around and manipulate into difference scenarios will help you to investigate the problem.

**Patterns**

Patterns are a sequence of repeated objects, shapes and numbers.

**Problem Solving Strategy 8** (Look for and Explain Patterns).

* Pattern recognition is key in problem solving!
* Recognizing a pattern helps break down the problem and builds a path to finding the solution.

**Problem Solving Strategy 9** (Find the Math, Remove the Context).

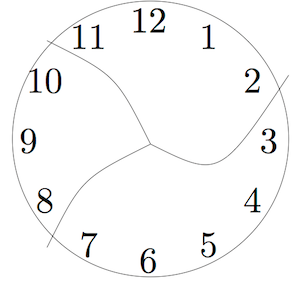
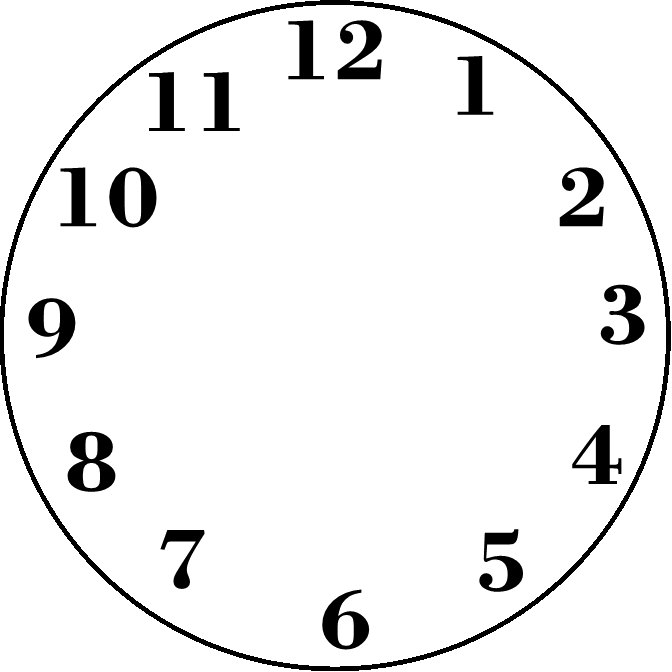
* Get rid of unimportant details.
* Find the underlying math problem.
* Go back to the original question and see if you can solve it using the math.

**Problem Solving Strategy 10** (Check Your Assumptions).

* Don’t add extra assumptions that are not in the problem.
* Ask yourself, “Am I constraining my thinking too much?”

**Use the Problem-Solving Strategies discussed to answer this problem?**

### Problem 4 (Broken Clock)

This clock has been broken into three pieces. If you add the numbers in each piece, the sums are consecutive numbers. (**Consecutive numbers** are whole numbers that appear one after the other, such as 1, 2, 3, 4 or 13, 14, 15.)

Can you break another clock into a different number of pieces so that the sums are consecutive numbers? Assume that each piece has at least two numbers and that no number is damaged (e.g. 12 isn’t split into two digits 1 and 2.)

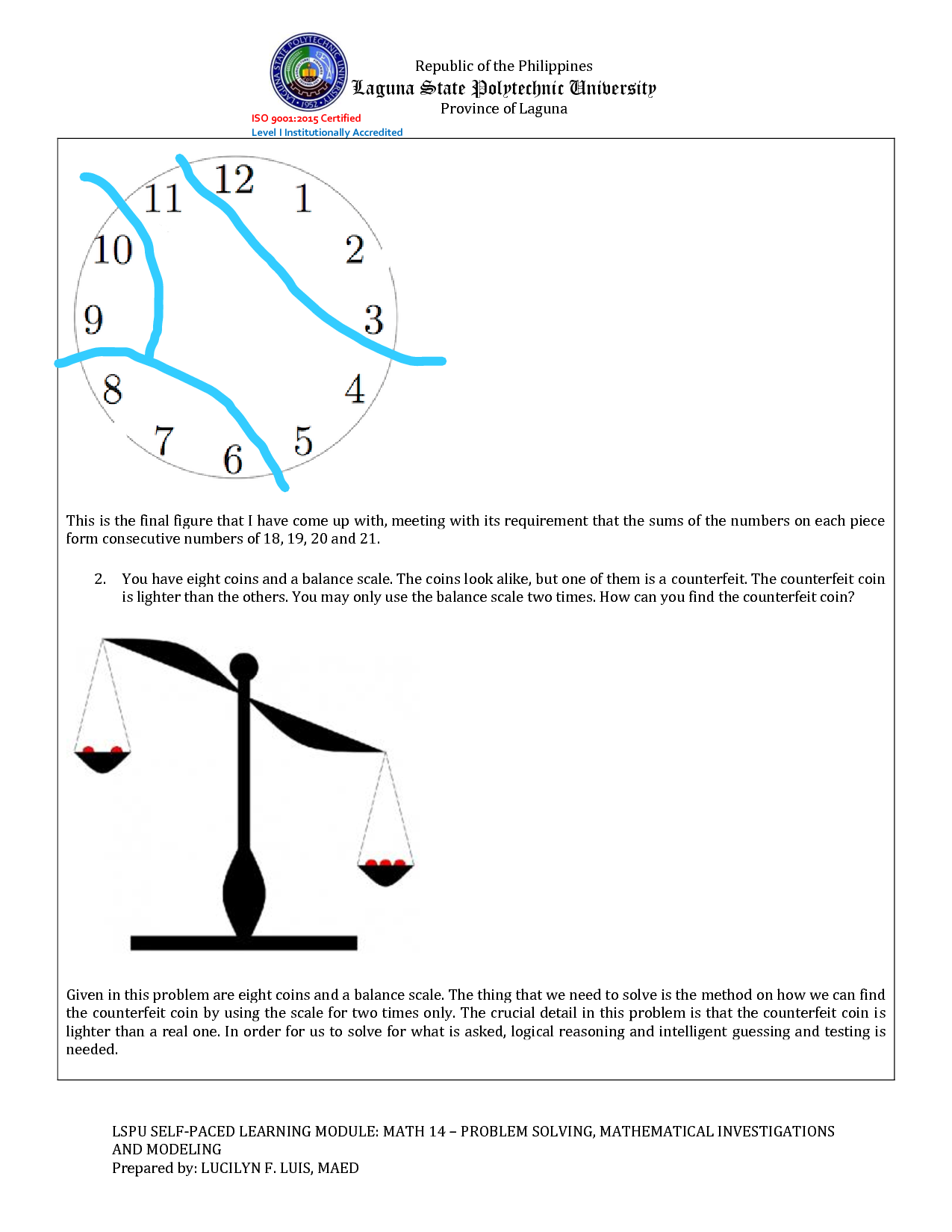
18, 19, 20, 21

What are the sums of the numbers on each piece? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Yes

Are they consecutive? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Answer!



18

20

19

21