**Lesson 1 Notes: Problem Solving Strategies**

*The primary goal of this class is to challenge you to think like a Mathematician!*

The Common Core State Standards for Mathematics identifies eight “Mathematical Practices.” The first practice is:

Part of solving a problem is:

* Understanding what is being asked
* Knowing what a solution should look like

**Problem or Exercise?**

***Problems often involve false starts, making mistakes, and lots of scratch paper!***

**For each question below, decide if it is a**problem**or an**exercise**. (You do not need to solve the problems! Decide which category fits for you.)**

**Problem Solving Strategies**

1. What is the sum of 125 and 30?

Problem or Exercise: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2. This clock has been broken into three pieces. If you add the numbers in each piece, the sums are consecutive numbers. (Note: Consecutive numbers are whole numbers that appear one after the other, such as 1, 2, 3, 4 or 13, 14, 15.)

Problem or Exercise: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

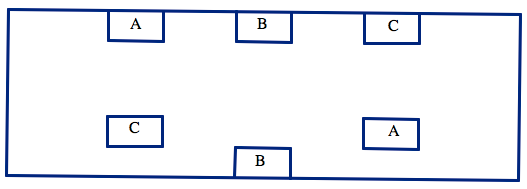
### Problem or Exercise: \_\_ Problem\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Polya’s Four-Step Method for Problem Solving**

**Problem Solving Strategy 1** (Wishful Thinking)

**Problem 1 (ABC)**

Draw curves connecting A to A, B to B, and C to C.  Your curves cannot cross or even touch each other. They cannot cross through any of the lettered boxes, and they cannot go outside the large box or even touch its sides.



Do not be afraid to change the problem! Ask yourself “what if” questions:

* What if the picture was different?
* What if the numbers were simpler?
* What if I just made up some numbers?

[**Watch Video!**](https://www.youtube.com/watch?v=aoZ0qSruJC0&t=9s)

**The Most Important Problem-Solving Strategy**

**Problem Solving Strategy 3** (Draw a Picture).

### Problem 2 (Payback)

Last week, Alex borrowed money from several of his friends. He finally got paid at work, so he brought cash to school to pay back his debts. First, he saw Brianna, and he gave her 1/4 of the money he had brought to school. Then Alex saw Chris and gave him 1/3 of what he had left after paying Brianna. Finally, Alex saw David and gave him 1/2 of what he had remaining. Who got the most money from Alex?

*Can you represent something in the situation by a picture?*[**Watch Video**](https://youtu.be/xCEVafhcdSk)

**Problem Solving Strategy 4** (Make Up Numbers).  You can work forwards: Assume Alex had some specific amount of money when he showed up at school, say $100. Then figure out how much he gives to each person. Or you can work backwards: suppose he has some specific amount left at the end, like $10. Since he gave David half of what he had left, that means he had $20 before running into David. Now, work backwards and figure out how much each person got. What’s the answer?

**Start with a fixed amount**

**Work Backwards**

### Problem 3 (Squares on a Chess Board)

How many squares, of any possible size, are on an 8 × 8 chess board? (The answer is not 64… It’s a lot bigger!)

**Explain Problem Solving Strategy 5** (Try a Simpler Problem) using the above problem.

**Problem Solving Strategy 6** (Work Systematically).

For example, in this problem you might keep track of how many 1 × 1 squares are on each board, how many 2 × 2 squares on are each board, how many 3 × 3 squares are on each board, and so on. You could keep track of the information in a table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **size of board** | **# of 1 × 1 squares** | **# of 2 × 2 squares** | **# of 3 × 3 squares** | **# of 4 × 4 squares** | **…** |
| **1 by 1** | 1 | 0 | 0 | 0 |  |
| **2 by 2** | 4 | 1 | 0 | 0 |  |
| **3 by 3** | 9 | 4 | 1 | 0 |  |

**Try it for an 8 X 8 chessboard?**

**What if we were trying to figure out the number of possible squares for a 3 X 3 chessboard?**

***“It’s not that I’m so smart; it’s just that I stay with problems longer.” (Albert Einstein)***

Even Albert Einstein was wrong! But, the secret to being a good mathematician is to keep trying.

**Manipulatives**

**Problem Solving Strategy 7** (Use Manipulatives to Help You Investigate).

**Patterns**

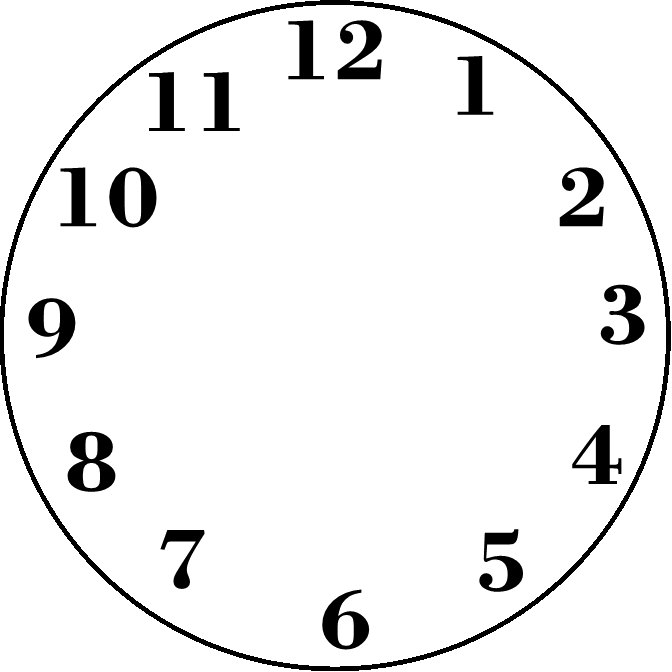
**Problem Solving Strategy 8** (Look for and Explain Patterns).

**Problem Solving Strategy 9** (Find the Math, Remove the Context).

**Problem Solving Strategy 10** (Check Your Assumptions).

**Use the Problem-Solving Strategies discussed to answer this problem?**

### Problem 4 (Broken Clock)

A clock with numbers on it. The clock is broken into three segments as follows: (11, 12, 1, 2), (3, 4, 5, 6, 7), and (8, 9, 10). 
This clock has been broken into three pieces. If you add the numbers in each piece, the sums are consecutive numbers. (**Consecutive numbers** are whole numbers that appear one after the other, such as 1, 2, 3, 4 or 13, 14, 15.)

Can you break another clock into a different number of pieces so that the sums are consecutive numbers? Assume that each piece has at least two numbers and that no number is damaged (e.g. 12 isn’t split into two digits 1 and 2.)

What are the sums of the numbers on each piece? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Are they consecutive? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_