**Lesson 4 Notes: Logic**

**ANSWER KEY**

*Logic prepares students for life, and it starts in your classroom. It teaches them reasoning skills they need to navigate life. It’s more than math!*

**Statement (Proposition)**

A statement is a declarative sentence that is either true or false but not both.

Example: Atlanta is the capital of Georgia. (yes)

Example: Pizza is delicious. (no)

**Compound statement**

A compound statement is formed by joining two or more statements together.

Example: The snow is falling AND it is cold.

**Negation**

Negation of a statement p means not p (˜P)

**Example 1**

***Negate each of the following statements.***

1. 2 + 3 = 6 2. A triangle has four sides.

2 + 3 ≠ 6 A triangle does not have four sides.

***Truth tables are used to test the validity of an argument!***

***Statement: Math is fun!***

Be careful! A statement can be proven true or false. Only Math teachers can prove this statement. 😊

**Truth Tables**

A truth table is a mathematical table used to determine if a compound statement is true or false.

|  |  |
| --- | --- |
| NEGATION | |
| P | **˜P** |
| T | **F** |
| F | **T** |

There are three basic logical connectives we use in determining the validity of a truth table. (And, Or and Not)

**Fill in the truth table for Conjunction*.***

**Disjunction**

 A disjunction is a compound statement formed by joining two statements with the connector OR.

We use this symbol to represent conjunction in a truth table. \_\_\_\_\_\_\_\_\_\_

The truth value of any compound statement, such as p ˅ q is defined using the truth value of each statement. A disjunction of p and q is defined to be true if and one or both p and q are \_\_\_\_\_\_\_\_\_\_\_\_. Otherwise, it is false.

TRUE

TRUE

˅

**Conjunction**

A conjunction is a type of connective that uses the word “and” to join together two propositions.

We use this symbol to represent conjunction in a truth table. \_\_\_\_\_\_\_\_\_\_

The truth value of any compound statement, such as p ^ q is defined using the truth value of each statement.

A conjunction of p and q is defined to be true, if and only if, both p and q are \_\_\_\_\_\_\_\_\_\_\_\_. Otherwise, it is false.

∧

|  |  |  |
| --- | --- | --- |
| ***p*** | ***q*** | ***p ^ q*** |
| ***T*** | ***T*** | *T* |
| ***T*** | ***F*** | *F* |
| ***F*** | ***T*** | *F* |
| ***F*** | ***F*** | *F* |

***Fill in the truth table for Disjunction.***

|  |  |  |
| --- | --- | --- |
| ***p*** | ***q*** | ***p ˅ q*** |
| ***T*** | ***T*** | *T* |
| ***T*** | ***F*** | *T* |
| ***F*** | ***T*** | *T* |
| ***F*** | ***F*** | *F* |

***Classify each of the following statements as true or false.***

***Let p be 4 + 3 = 7, let q be 2 \* 5 = 10 and let r be 6 + 3 = 8***

1. p ^ q b. ˜p ˅ r c. q ˅ r d. ˜(p ˄ q)

F

F

T

T

***Show that ˜(p ˄ q) = ˜p ˅ ˜q***

hypothesis

conclusion

p q

**Conditional Statement**

A conditional statement is expressed in the form of “if p then q” where p and q are statements.

We use this symbol to represent the conditional statement in a truth table. \_\_\_\_\_\_\_\_\_\_.

The “if” part of a conditional is the \_\_\_\_\_\_\_\_\_\_\_\_\_ and the “then” part is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

"If p, then q" is true when p is false and q is true.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ***p*** | ***q*** | ***p ˄ q*** | ***˜(p ˄ q)*** | ***˜p*** | ***˜q*** | ***˜p ˅ ˜q*** |
| ***T*** | ***T*** | *T* | *F* | *F* | *F* | *F* |
| ***T*** | ***F*** | *F* | *T* | *F* | *T* | *T* |
| ***F*** | ***T*** | *F* | *T* | *T* | *F* | *T* |
| ***F*** | ***F*** | *F* | *T* | *T* | *T* | *T* |

***Fill in the truth table for the*** p → q.

|  |  |  |
| --- | --- | --- |
| ***p*** | ***q*** | *p → q* |
| ***T*** | ***T*** | *F* |
| ***T*** | ***F*** | *F* |
| ***F*** | ***T*** | *T* |
| ***F*** | ***F*** | *F* |

*(Hint: The easiest way to remember this truth table is to remember the only time the if then conditional is true is when p is false and q is true)!*

**Example 2: Write the converse, inverse, and the contrapositive for the following statement:**

**Statement:** *if p, then q*p → q

**Converse***: if q, then p* q → p

**Inverse:** *if not p, then not q* ~p → ~q

**Contrapositive:** *if not q, then not p*~q → ~p

*If I am in Douglas, then I am in Georgia.*

**Converse:** If I am in Georgia, then I am in Douglas.

**Inverse:** If I am not in Douglas, then I am not in Georgia.

**Contrapositive:** If I am not in Georgia, then I am not in Douglas.

Logically equivalent

**A statement and its contrapositive are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.**

**Example 3: Identify the converse, inverse and contrapositive of the following statement:**

*If it is raining, then the grass is wet.*

inverse

1. If it is not raining, then the grass is not wet. **\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

converse

2. If the grass is wet, then it is raining. **\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

contrapositive

3. If the grass is not wet, then it is not raining. **\_\_\_\_\_\_\_\_\_\_\_\_**

Valid argument

I

**Valid Argument**

 An argument is valid if and only if it is necessary that if all of the premises are true, then the conclusion is true.

Hypothesis: In Atlanta, all lobbyist has influence.

No one in Atlanta is over 5 ft tall.

Conclusion: Persons over 5 ft tall are not lobbyist in Atlanta.